Monetary policy spillovers, global commodity prices and cooperation*

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Abstract

How are the monetary policy trade-offs when central banks misdiagnose commodity price movements as coming from global demand shocks versus external supply shocks? To analyse this question we present a framework of a three-country (zones) model with one commodity importer, a dominant exporting country and a fringe of competitive exporting countries, in which commodity prices are endogenously determined. We examine the effects of monetary policy responses to global commodity price developments. This new framework is also used to analyse the dynamics of monetary policy spillovers on global commodity prices, the effects of misperceptions of the source of these commodity price fluctuations, and the potential costs and benefits of monetary policy cooperation in such a globalised world. The main results are the following: i) in contrast to the exogenous commodity price case, a monetary policy trade-off arises in the commodity importer country from commodity supply shocks when prices are endogenous. That is, it becomes optimal for the monetary authority to stabilise partially the effects of fluctuations in commodity prices on headline inflation. ii) The optimal response to demand shocks is to fully lean against the wind. iii) If the central bank cannot identify the source of the shocks driving commodity price fluctuations, it would exacerbate the volatility of inflation, output and commodity prices.

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1 Introduction

Over the past ten years, global commodity prices have experienced wide swings, with some commodity prices reaching historically high levels in the run-up before the Great Financial Crisis, to then plummet as the recession burst. Policymakers are not new to commodity price volatility: while most commodity prices remained broadly stable during the so-called "Great Moderation", they displayed wide swings in the 70s, amid geopolitical tensions that pushed the oil price to unprecedented levels.

So, it is no surprise that the conventional wisdom about the optimal monetary policy response to commodity price fluctuations was built largely with the experience of the 70s in mind. Commodity price fluctuations were interpreted as the result of exogenous shocks; this implies that each country should respond only to those fluctuations that lead to second-round inflationary effects. In practice, this accounts to a strong focus on core inflation, which allows to look through the initial impact of commodity price fluctuations on headline inflation.

However, the broad-based surge in commodity prices in of the late 2000s, and the subsequent fall in the wake of the Great Recession led many to reconsider the drivers of commodity price fluctuations. A large consensus emerged that the recent fluctuations in commodity prices are to be largely ascribed to buoyant global demand, in particular from commodity-hungry emerging economies (Kilian 2009). Yet, global demand is likely to be influenced by monetary conditions: for example, Anzuini et al (2013) report evidence that loose monetary policy has an impact on commodity prices via the demand channel. If this is so, any prescription regarding optimal monetary policy cannot abstract from the endogeneity of commodity prices. More specifically, it is important that monetary authorities are able to distinguish the source of commodity price fluctuations in order to deploy an optimal response.

The endogeneity of commodity prices to monetary policy also raises issues of cooperation: while a single country could be sufficiently small not to be able to influence prices with its own monetary policy, if all countries follow the same strategy, a sub-optimal equilibrium will result at the global level.

Yet, the bulk of the theoretical literature has so far stayed clear of models with endogenous commodity prices (see eg Leduc and Sill 2004, Carlstom and Fuerst 2006, Montoro 2012, Natal 2012, Catao and Chang 2014). So, the common knowledge on the optimal monetary policy response to commodity price fluctuations was largely built on the notion of exogenous commodity price shocks. This stream of literature focused mainly on whether it is optimal to target core or headline inflation, or whether there is a trade-off between stabilising output and controlling inflation. However, in such a framework, the optimal response is obviously independent on the source of the shock: as spelled out explicitly by Blanchard and Gali (2010) an increase in commodity prices which is driven by foreign demand is to be treated as a supply shock.

However, endogenous commodity prices are necessary if one wants to differentiate the optimal response to different sources of commodity price fluctuations. Another stream of theoretical papers tried indeed to endogenise commodity prices into small-scale DSGE models (Backus and Crucini 1998, Bodenstein, Erceg and Guerrieri 2011, Nakov and Nuno 2011). Still, such papers have ignored monetary policy, focusing more on the determination of the price of oil and the frictions affecting it. Nakov and Pescatori (2010) is probably the first attempt to include monetary policy in a model in which commodity prices are determined endogenously. Another recent contribution in this direction is Bodenstein, Guerrieri and Kilian (2012), which report direct evidence that the optimal monetary policy response crucially depends on the source of the shock.

In our model commodity prices are endogenously determined by the equilibrium between global demand from importers countries and global supply from two types of exporters: competitive and monopolistic. Commodities are used for both consumption and production in the importing country. We derive the optimal monetary policy response to commodity price fluctuations, differentiating it by the driver of such fluctuations, and also analyse the case when the central bank cannot properly disentangle those sources of commodity price fluctuations. Moreover, we also plan to extend the model to have more than one commodity importing country to analyse the effects of cross-border monetary policy spillovers through the commodities market and the role for policy cooperation.

The main results from our model are: first, different from the case when commodity prices are considered as exogenous, which implies core inflation stabilisation as the optimal policy, the divine coincidence between inflation and output gap stabilisation is broken when we take into account endogenous fluctuations in commodity prices. Second, as in the standard New Keynesian closed economy model, the optimal response to demand shocks is to fully lean against the wind, neutralising all the effects on commodity prices from domestic demand shocks. Third, the optimal response to negative commodity supply shocks, which increase commodity prices, is to generate negative core inflation such it partially offsets the increase in headline inflation. Fourth, when commodity price fluctuations are driven by demand shocks, but the central bank incorrectly confuses it with external supply shocks, it would fail to stabilise aggregate demand. This policy misperception would generate fluctuations in the commodity prices that otherwise would have been able to stabilise. Fifth, in the opposite case, when the monetary authority confuses supply driven increase in commodity prices with a demand driven one, the contraction in both output and core inflation would be higher than the optimal. These results shed light on the importance of correctly identifying the sources of fluctuations in commodity prices in the design of optimal monetary policy.

2 The Model

We present a global economic model in which commodity prices are determined endogenously, in the spirit of Nakov and Pescatori (2010). In this setup there are two large regions: a commodity importing and a commodity exporting one. The latter is composed by a dominant commodity exporter country and a fringe of small competitive commodity exporting countries.

In the basic setup, the commodity importing region is composed by only one country, but this can extended later to a group of importing countries. The commodity importers do not produce the commodity itself, but produce final goods such that they are the only exporters. The commodity goods are used as both consumption good and input for production. There is no capital. Final goods producers are subject to monopolistic competition and nominal rigidities, and the central bank sets monetary policy using a Taylor rule. To have the simplest possible setup we assume there is no capital accumulation.

The commodity exporting countries produce only commodity goods, using as input a fraction of the final goods sold by the importing region. In addition, they buy final goods from the importing countries for consumption. We assume that nominal rigidities are absent in the exporting countries. The dominant commodity exporting country has market power and sets prices above marginal costs. On the other hand, the competitive exporting countries are similar in structure to the dominant exporting country, but operate in perfect competition taking commodity prices as given.

Also, we further assume cross-border financial autarky: there is no borrowing across regions and the regional current accounts are balanced in each period. Trade is carried out in a common world currency, so exchange rates are not defined.

This model can be modified in different fronts to be more realistic, for example including final goods production and nominal rigidities in the exporting countries. However, at this stage the model is maintained as simple as possible to analyse the implications of the importing countries' monetary policy on the commodity price from a global perspective.

2.1 Commodity importing country

2.1.1 Households

We assume the following utility function on consumption and labour of the representative consumer

$$U_{t_o} = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \exp(g_t) \left[\ln(C_t) - \frac{L_t^{1+v}}{1+v} \right], \tag{2.1}$$

where g_t is a preference shock and v captures the inverse of the elasticity of labour supply. The consumption basket is defined as a Cobb-Douglas aggregator of the final goods consumption basket $C_{Y,t}$ and the household's demand for commodities $O_{C,t}$:

$$C_t = (C_{Y,t})^{1-\gamma} (O_{C,t})^{\gamma}. \tag{2.2}$$

The consumption of final goods $C_{Y,t}$ is a Dixit-Stiglitz aggregate of a continuum of differentiated goods $C_{Y,t}(z)$:

$$C_{Y,t} = \left[\int_0^1 C_{Y,t}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2.3}$$

The optimising household takes decisions subject to a standard budget constraint which is given by

$$C_t = \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t}\frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t}, \tag{2.4}$$

where W_t is the nominal wage, P_t is the price of the consumption good, B_t is the end of period nominal bond holdings, R_t is the riskless nominal gross interest rate , Γ_t is the share of the representative household on total nominal profits, and T_t are net transfers from the government. The first order conditions for the optimising consumer's problem are:

$$1 = \beta E_t \left[R_t \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-1} \exp(g_{t+1} - g_t) \right], \tag{2.5}$$

$$\frac{W_t}{P_t} = C_t L_t^v, \tag{2.6}$$

$$\begin{aligned} O_{C,t} &= \gamma \frac{P_t}{P_{Ot}} C_t, & (2.7) \\ C_{Y,t} &= (1-\gamma) \frac{P_t}{P_{Yt}} C_t, & (2.8) \\ C_{Y,t}(z) &= \left(\frac{P_{Y,t}(z)}{P_{Y,t}} \right)^{-\varepsilon} C_{Y,t}. & (2.9) \end{aligned}$$

Equation (2.5) is the standard Euler equation that determines the optimal path of consumption. Equation (2.6) describes the optimal labour supply decision. Equations (2.7), (2.8) and (2.9) are the relative demand for commodities, final goods, and final good z in the consumption basket.

Replacing (2.7) and (2.8) in (2.2) defines the price level and headline inflation:

$$\begin{aligned} P_t &= (P_{Y,t})^{1-\gamma} (P_{O,t})^{\gamma}, & (2.10) \\ \Pi_t &= (\Pi_{Y,t})^{1-\gamma} (\Pi_{O,t})^{\gamma}, & (2.11) \end{aligned}$$

where $\Pi_t = P_t/P_{t-1}$, $\Pi_{Y,t} = P_{Y,t}/P_{Y,t-1}$ and $\Pi_{O,t} = P_{O,t}/P_{O,t-1}$ are headline, core and non-core/commodity inflation.

Similarly, replacing (2.9) in (2.3) defines the price level of final goods

$$P_{Y,t} = \left[\int_0^1 P_{Y,t}(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}. \tag{2.12}$$

2.1.2 Firms goods producers

Final goods are produced under monopolistic competition using the following Cobb-Douglas technology:

$$Y_t(z) = A_t L_t(z)^{1-\alpha} O_{Y,t}(z)^{\alpha}, \tag{2.13}$$

where O_{Yt} is the bundle of commodities used as input, and α denotes the share of commodities in the production function. The real commodity price, $Q_t = \frac{P_{O,t}}{P_t}$, is determined in the world market. Note that from equation (2.10), $Q_t = (P_{O,t}/P_{Y,t})^{1-\gamma}$ is proportional to the inverse of terms of trade of the importing country.

From the cost minimisation problem of the firm we obtain an expression for the real marginal cost:

$$MC_t(z) = \left(\frac{W_t}{P_t} \right)^{1-\alpha} (Q_t)^{\alpha} / \left[A_t (1-\alpha)^{1-\alpha} \alpha^{\alpha} \right], \tag{2.14}$$

where $MC_t(z)$ are real marginal cost. Notice that real marginal costs are the same for all final good firms, since technology has constant returns to scale and factor markets are competitive, i.e. $MC_t(z) = MC_t$. On the other hand, the first order condition for final goods producers with respect to labour and commodities imply the following input demands for the individual firm:

$$L_t(z) = (1-\alpha)\frac{MC_t}{W_t/P_t}Y_t(z), \tag{2.15}$$

$$O_{Y,t}(z) = \alpha\frac{MC_t}{Q_t}Y_t(z). \tag{2.16}$$

From (2.9), the individual demand for each final goods is:

$$Y_t(z) = \left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\varepsilon} Y_t, \tag{2.17}$$

where Y_t is the aggregate demand. Final goods producers set prices following a staggered pricing mechanism *a la Calvo*. Each firm faces an exogenous probability of changing prices given by $(1-\theta)$. A firm that changes its price in period t chooses its new price $P_{Y,t}(z)$ to maximise:

$$E_t \sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t+k}^{-1} \Gamma_{t+k}(z),$$

where $\zeta_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-1}$ is the stochastic discount factor, and $F_{t+k} = \frac{P_{Y,t+k}}{P_{Y,t}}$ the cumulative level of core inflation. The function: $\Gamma_t(z) = [(1-\tau)P_{Y,t}(z) - P_t MC_t]\left(\frac{P_{Y,t}(z)}{P_{Y,t}}\right)^{-\varepsilon} Y_t$ is the after-tax nominal profits of the supplier of good z with price $P_{Y,t}(z)$. τ is the proportional tax on sale revenues which we assume constant.

The optimal price that solves the firm's problem is given by

$$\left(\frac{\widetilde{P}_{Y,t}(z)}{P_{Y,t}}\right) = \frac{\mu E_t\left[\sum\limits_{k=0}^{\infty}\theta^k\zeta_{t,t+k}MC_{t,t+k}F_{t+k}^{\varepsilon}Y_{t+k}\right]}{E_t\left[\sum\limits_{k=0}^{\infty}\theta^k\zeta_{t,t+k}F_{t+k}^{\varepsilon-1}Y_{t+k}\right]}, \tag{2.18}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon-1}/(1-\tau)$ is the final goods price markup net of taxes and $\widetilde{P}_{Y,t}(z)$ is the optimal price level chosen by firm z.

Since only a fraction $(1-\theta)$ of firms changes prices every period and the remaining one keeps its price fixed, the aggregate price level, defined as the price of the final good that minimise the cost of the final goods producers, is given by the following equation:

$$P_{Y,t}^{1-\varepsilon} = \theta P_{Y,t-1}^{1-\varepsilon} + (1-\theta)\left[\widetilde{P}_{Y,t}(z)\right]^{1-\varepsilon}. \tag{2.19}$$

Following Benigno and Woodford (2005), equations (2.18) and (2.19) can be written recursively introducing some auxiliary variables N_t and D_t:

$$\theta\left(\Pi_{Y,t}\right)^{\varepsilon-1} = 1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\epsilon}, \tag{2.20}$$

$$D_t = Y_t/C_t + \theta\beta E_t \left[(\Pi_{Y,t+1})^{\epsilon-1} D_{t+1} \right], \tag{2.21}$$

$$N_t = \mu Y_t MC_t/C_t + \theta\beta E_t \left[(\Pi_{Y,t+1})^{\epsilon} N_{t+1} \right]. \tag{2.22}$$

Equation (2.20) comes from the aggregation of individual firms prices. The ratio N_t/D_t represents the optimal relative price $\widetilde{P}_{Y,t}(z)/P_{Y,t}$. These three last equations summarise the recursive representation of the non linear Phillips curve for non-commodity goods, that is core inflation.

2.1.3 Government and monetary policy

Government transfers to households the proceeds from taxing final good producers:

$$T_t = \tau P_{Y,t} Y_t. \tag{2.23}$$

Also, we assume the central bank can set the interest rate according to different specifications of a Taylor rule. The different policy alternatives are described in the next section.

2.1.4 Aggregation

Aggregating the demand for labour, commodities and final goods yields:

$$L_t = \int_0^1 L_t(z)dz, \tag{2.24}$$

$$O_{Y,t} = \int_0^1 O_{Y,t}(z)dz, \tag{2.25}$$

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2.26}$$

The above, together with (2.15), (2.16), (2.13) and (2.17) imply:

$$L_t = (1-\alpha) \frac{MC_t}{W_t/P_t} Y_t \Delta_t, \tag{2.27}$$

$$O_{Y,t} = \alpha \frac{MC_t}{Q_t} Y_t \Delta_t, \tag{2.28}$$

$$Y_t = A_t L_t^{1-\alpha} O_{Y,t}^{\alpha} / \Delta_t. \tag{2.29}$$

where $\Delta_t = \int_0^1 \left(\frac{P_{Y,t}(z)}{P_{Y,t}} \right)^{-\varepsilon} dz$ is a measure of price dispersion. Since relative prices differ across firms due to staggered price setting, input usage will differ as well. Therefore, the price dispersion factor, Δ_t, appears in the aggregate input demands equations. We use (2.19) to derive the law of motion of Δ_t :

$$\Delta_t = (1-\theta) \left(\frac{1-\theta (\Pi_{Y,t})^{\varepsilon-1}}{1-\theta} \right)^{\varepsilon/(\varepsilon-1)} + \theta \Delta_{t-1} (\Pi_{Y,t})^{\varepsilon}. \tag{2.30}$$

We can see, from (2.30), that higher inflation increases price dispersion and, from (2.27), that higher price dispersion increases the labour amount necessary to produce a certain level of output, implying more disutility in (2.1).

2.1.5 Market clearing

Since we have cross-border financial autarky, bonds are in zero net supply:

$$B_t = 0. \tag{2.31}$$

Substituting this and aggregating profits in the budget constraint of the household defines the aggregate resources constraint:

$$\frac{P_{Yt}}{P_t} C_{Yt} = \frac{P_{Yt}}{P_t} Y_t - Q_t \left(O_t + X_t \right), \tag{2.32}$$

where O_t and X_t are the aggregate supply of commodities by the dominant and the competitive fringe of exporting countries, respectively.

Total aggregate demand for commodities for both consumption and production equals its aggregate supply:

$$O_{Ct} + O_{Yt} = O_t + X_t. \tag{2.33}$$

Taking into account the latter and the consumption bundles (2.7) and (2.8), the aggregate resources constraints become:

$$\frac{P_{Yt}}{P_t} Y_t = C_t + Q_t O_{Yt} \tag{2.34}$$

Also, after replacing the budget constraints of the commodity exporters defined below, the aggregate demand for final goods can be written as:

$$Y_t = C_{Yt} + C_t^* + I_t^* + C_t^{*,c} + I_t^{*,c}. \tag{2.35}$$

where the four last terms on the right-hand-side are the demand of final goods for consumption and investment by the dominant and the competitive fringe of exporting countries, respectively.

2.2 Commodity exporting countries

The commodity industry is modelled as a combination of a dominant exporter and a group of competitive exporters. The relative share of the non-competitive market allows us to analyse different market structures between perfect competition and a single monopolist. Under this setup, monetary policy in the importing country has spillovers on the commodity market, through its impact on the markups of the commodity goods. For this transmission channel the assumption of imperfect competition in the commodity market is key.

2.2.1 Dominant commodity exporter

The exporting country produces commodities according to:

$$O_t = Z_t I_t^*, \tag{2.36}$$

where Z_t is an exogenous productivity shifter, and I_t^* is an intermediate good used in commodity production which is bought from the commodity importing country. The productivity evolves exogenously according to

$$\ln Z_t = (1 - \rho_z) \ln \overline{Z} + \rho_z \ln Z_{t-1} + \varepsilon_t^z, \tag{2.37}$$

where $\varepsilon_t^z \sim i.i.d.N\left(0, \sigma_z^2\right)$.

The utility function of the households in the dominant commodity exporter country depends only on consumption, since they do not face labour decissions:

$$U_{t_o}^* = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \ln\left(C_t^*\right). \tag{2.38}$$

The households face the following period budget constraint:

$$P_{Y,t} C_t^* = \Gamma_t^*, \tag{2.39}$$

which equates consumption expenditure to dividends from commodity production, Γ_t^*, that is wholly owned by the household. As such, the representative household's objective of expected utility maximisation is consistent with maximising the expected present discounted value of the logarithm of real profits from commodity production. Real profits each period are given by

$$\frac{\Gamma_t^*}{P_t} = Q_t O_t - \frac{P_{Y,t}}{P_t} I_t^*. \tag{2.40}$$

The consumption good C_t^* and the intermediate good I_t^* are final goods as those in the non-commodity consumption basket of the commodity importing country. The commodity exporter chooses the level of commodity output (O_t), such it maximises the expected present discounted utility of the representative household (2.38), subject to the demand from the importing country (2.33) and the supply of the competitive fringe of small exporters (X_t).

2.2.2 The fringe of small competitive commodity exporters

We model the fringe of competitive commodity exporters as in Nakov and Pescatori (2010). Apart from the dominant commodity exporter, in the rest of the world there is a continuum of atomistic commodity firms indexed by $j \in [0, \Omega_t]$. Each country produces a quantity $X_t(j)$ of the commodity according to the technology

$$X_t(j) = \xi(j) Z_t I_t^{*,c}(j)$$

subject to the capacity constraint

$$X_t(j) = \in \left[0, \overline{X}\right]$$

where $[\xi(j) Z_t]^{-1}$ is the marginal cost of commodity production of country j, that depends on a idiosicratic and aggregate component. The input $I_t^{*,c}(j)$ is an intermediate good used in commodity production and also bought from the commodity importing country.

The production of the commodity can be sold at the international real price Q_t, which the atomistic exporters take as given, or it is lost. Each country chooses the amount of commodity to produce each period so as to maximise profits:

$$\max\left[Q_t X_t(j) - \frac{\frac{P_{Y,t}}{P_t} X_t(j)}{\xi(j) Z_t}\right]$$

s.t.

$$X_t(j) = \in \left[0, \overline{X}\right]$$

We assume the idiosincratic component $1/\xi(j)$ has a uniform distribution $F\left[1/\xi(j)\right]$ in the interval a and b. The total mass of competitive fringe countries is Ω_t. Then, the amount of commodities produced by the competitive fringe as a whole is given by:

$$X_t \equiv \int_0^{\Omega_t} X_t(j)\, d_j = \Omega_t F(Q_t Z_t)$$

Then, total production is given by:

$$X_t = \begin{cases} \Omega_t \overline{X} & Q_t Z_t > b \\ \Omega_t \overline{X} \frac{Q_t Z_t - a}{b-a}, & a < Q_t Z_t \leq b \\ 0 & Q_t Z_t \geq a \end{cases}$$

If we further assume that $a = 0$ and normalise $b = \overline{X} > 1$, such that is sufficiently large that at least some competitive fringe producers are always priced out of the market by the dominant country. With these assumptions, the supply by the fringe of competitive exporters is:

$$X_t = \Omega_t Z_t Q_t \tag{2.41}$$

2.2.3 The dominant commodity exporter problem

In the case that the commodities market had perfect competition, commodity prices are equal to marginal costs:

$$Q_t^{PC} = Z_t^{-1} \tag{2.42}$$

and the produced quantity of commodities is given by the global demand at such price.

On the other hand, under imperfect competition, the commodity exporter maximises the expected present discounted value of the logarithm of commodity profits

$$\max E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \ln\left(Q_t O_t - O_t/Z_t\right), \tag{2.43}$$

subject to global demand for commodities and the supply of the competitive fringe

$$O_t = O_{C,t} + O_{Y,t} - X_t, \tag{2.44}$$

which is equal to:

$$O_t = \gamma \frac{1}{Q_t} C_t + \alpha \frac{MC_t}{Q_t} Y_t \Delta_t - \Omega_t Q_t Z_t \tag{2.45}$$

after replacing the demand for commodities, (2.7) and (2.28), and the supply from the competitive exporters (2.41) in (2.44). We assume the dominant commodity exporter takes as given the macroeconomic variables of the importing country (C_t, MC_t, Y_t and Δ_t). That is, we are assuming that it does not internalise the effects of its actions in the macroeconomic performance of the importing country[1].

The first order condition of this problem gives the price of commodity in real terms (details in Appendix D):

$$Q_t^{NI} = \Psi_t Z_t^{-1}, \tag{2.46}$$

where $\Psi_t \equiv \frac{1}{1-\eta_t}$ is the commodity market mark-up and $\eta_t = O_t/\left(O_t + 2X_t\right)$ is the elasticity of substitution of the net global demand for commodities in absolut value. According to this expression, the commodity price is a markup over its marginal costs. Supply shocks in the commodity exporting country affect the commodity price, and shocks in the commodity importing country have effects on the commodity price through the mark-up. The commodity price mark-up $\Psi_t \equiv 1 + \frac{O_t}{2X_t}$ is an increasing function of the supply of the dominant commodity exporter relative to supply of the competitive exporters. The limiting case when $O_t \to 0$ corresponds to perfect competition, whilst when $X_t \to 0$ is the case of a single monopolist.

3 Characterising optimal monetary policy

3.1 The benchmark output gap

We need to characterise the equilibrium allocation for some benchmarks scenarios to analyse the optimal monetary policy in the importing country. For this, we first express the level of output in terms of some variables of interest that account for the macroeconomic distortions. We use the labour demand (2.27), the labour supply (2.5), the aggregate demand (2.34), the commodity demand for production (2.28), and the aggregate production function (2.29) equations to solve for the level of output in terms of marginal costs, the price dispersion, the productivity level and the real commodity price[2]:

$$Y_t = \left(\frac{A_t}{\Delta_t}\right)^{1/(1-\alpha)} \left[\frac{(1-\alpha)\, MC_t\Delta_t}{(Q_t)^{-\gamma/(1-\gamma)} - \alpha MC_t\Delta_t}\right]^{1/(1+v)} \left(\alpha\frac{MC_t\Delta_t}{Q_t}\right)^{\alpha/(1-\alpha)}$$

The log linear approximation of the level of output in deviations from the steady state is equal to[3]:

$$y_t = \frac{1}{1-\alpha}a_t + \frac{\alpha}{1-\alpha}\left(mc_t - q_t\right) + \frac{1}{1+v}\Upsilon\left(mc_t + \frac{\gamma}{1-\gamma}q_t\right)$$

where $\Upsilon \equiv \left[1 - \frac{\alpha}{\mu}Q^{\gamma/(1-\gamma)}\right]^{-1} \geq 1$.

The natural output (y_t^n) is the one that is consistent with the equilibrium with flexible prices, such that $MC_t = \mu^{-1}$ is constant and $\Delta_t = 1$. In log linear terms, the natural output

[1] Nakov and Pescatori (2010) analyse the case in which the dominant commodity exporter internalise completely its actions in the importing country. That is, they solve for the Ramsey problem of the dominant commodity exporter taking into account as constraints the behavioural equations of the importer country.

[2] Derivations shown in appendix E

[3] Note that the linear approximation of the price dispersion (Δ_t) is not taken into account in this equation because it only has a second order effect, as shown by Benigno and Woodford (2005).

is equal to:

$$y_t^n = \frac{1}{1-\alpha} a_t - \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v} \frac{\gamma}{1-\gamma} \Upsilon \right) q_t \tag{3.1}$$

As shown in equation (3.1), commodity price fluctuations have two opposite effects on the natural output. When used in production - the α term - an increase in the commodity price has a similar effect than a negative productivity shock: it decreases the natural output. On the other hand, when used in consumption - the γ term - an increase in the commodity price increases the natural output. The latter is driven by an increase in labour because of a negative income effect on consumption from higher commodity prices.

The natural output gap ($\widehat{y}_t^n$), measured by the difference between the actual and the natural level of output, is then given by:

$$\widehat{y}_t^n = \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v} \Upsilon \right) mc_t \tag{3.2}$$

According to this, if the central bank responds to the natural output gap, it responds only to fluctuations in real marginal costs.

It is useful to analyse the equations that determine the dynamics of inflation to understand the importance of the output gap. As such, headline and core inflation are determined by the following two equations in log-linear terms:

$$\pi_t = \pi_{Y,t} + \frac{\gamma}{1-\gamma} \Delta q_t \tag{3.3}$$

$$\pi_{Y,t} = \kappa_y \widehat{y}_t^n + E_t \pi_{Y,t+1} \tag{3.4}$$

Equation (3.3) is the identity that defines headline inflation, while equation (3.4) is the Philips curve for final goods[4]. According to these expressions, stabilisation of the natural output gap is equivalent to stabilisation of core inflation. In that case, headline inflation would be proportional to the change in real commodity prices.

Similarly, we define the efficient level of output (y_t^e) as the one consistent with the efficient allocation, such that prices are flexible and there is no monopolistic distortions neither in the commodities market nor in the final goods market (that is, $Q_t^e = Z_t^{-1}$ and $\mu^e = 1$):

$$y_t^e = \frac{1}{1-\alpha} a_t + \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v} \frac{\gamma}{1-\gamma} \Upsilon^e \right) z_t \tag{3.5}$$

where $\Upsilon^e \equiv \left[1 - \alpha Z^{-\gamma/(1-\gamma)}\right]^{-1}$. The relationship between Υ^e and Υ depends on the size of the monopolistic distortions[5] and they are equal only if both markets are perfectly competitive or if commodities are not used for production (that is, $\alpha = 0$). The main difference with respect to natural output (3.1) is that only fluctuations associated to supply shocks in the commodity

[4]The equation of headline inflation comes from log-linearisation of equation (2.11) and the Phillips curve from log-linearisation of equations (2.20), (2.21) and (2.22), after replacing equation (3.2), where $\kappa_y \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} / \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v} \Upsilon \right)$.

[5]More precisely, $\Upsilon^e > (<) \, \Upsilon$ if $\Psi^{\gamma/(1-\gamma)} < (>) \, \mu$. Where Ψ and μ are the markups in steady state of the commodities and final goods market, respectivelly.

market affect the efficient output. That is, commodity mark-up fluctuations do not affect the efficient output. Then, a demand driven increase in the commodity price would leave the benchmark efficient output unaffected. On the other hand, a negative commodity supply shock would decrease both natural and efficient output levels, however at different magnitudes[6].

We define the efficient output gap ($\widehat{y}_t^e$) as the difference of actual output with respect to the efficient level of output:

$$\widehat{y}_t^e = \widehat{y}_t^n - \left(\frac{\alpha}{1-\alpha} - \frac{1}{1+v}\frac{\gamma}{1-\gamma}\Upsilon\right)\psi_t - \frac{1}{1+v}\frac{\gamma}{1-\gamma}\left(\Upsilon - \Upsilon^e\right) z_t \tag{3.6}$$

where ψ_t is the commodity market mark-up in log-linear deviations from the steady state. This is the welfare relevant output gap and it is equal to the natural output gap plus a term that depends on the commodity market mark-up and the commodity supply shock. Replacing equation (3.6) into (3.4) gives the following expression for the Phillips curve:

$$\pi_{Y,t} = \kappa_y \widehat{y}_t^e + E_t \pi_{Y,t+1} + u_t \tag{3.7}$$

where u_t is an endogenous cost-push shock, which is function of both ψ_t and z_t. According to this equation, if we take as a benchmark the efficient allocation, the divine coincidence is broken: in some cases it is not possible to stabilise both core inflation and the welfare relevant output gap. This trade-off arises from internalising the inefficient fluctuations in commodity prices. In particular, an increase in commodity price mark-ups generate a positive cost-push shock and pressures on core inflation.

3.2 The benchmark interest rate

As for the benchmark level of output, we find an expression for the benchmark interest rate replacing the aggregate resources constraint (2.34) and the definition of the price level (2.10) in the IS equation (2.5):

$$\frac{1}{R_t} = \beta E_t\left[\frac{1}{\Pi_{Y,t+1}}\left(\frac{1-\alpha MC_t Q_t^{\gamma/(1-\gamma)}\Delta_t}{1-\alpha MC_{t+1}Q_{t+1}^{\gamma/(1-\gamma)}\Delta_{t+1}}\right)\left(\frac{Y_t}{Y_{t+1}}\right)\exp\left(g_{t+1}-g_t\right)\right].$$

The **natural interest rate** is defined as the one consistent with flexible prices in the final goods, in which core inflation is zero. In log-linear terms the natural interest rate is equal to:

$$r_t^n = \left(g_t - E_t g_{t+1}\right) - \left(y_t^n - E_t y_{t+1}^n\right) + \frac{\gamma}{1-\gamma}\left(\Upsilon - 1\right)\left(q_t - E_t q_{t+1}\right) \tag{3.8}$$

Similarly, the **efficient interest rate** is calculated for the case when also the commodity and final goods markets are competitive:

$$r_t^e = \left(g_t - E_t g_{t+1}\right) - \left(y_t^e - E_t y_{t+1}^e\right) - \frac{\gamma}{1-\gamma}\left(\Upsilon^e - 1\right)\left(z_t - E_t z_{t+1}\right) \tag{3.9}$$

[6] In particular, in response to a negative commodity supply shock, the efficient output level contract less than the natural output, because in the latter the commodity mark-up partially offsets the effects of the supply shocks in the commodity price.

Both the natural and the efficient interest rate respond in the same way to the preference demand shocks, fully leaning against the wind and neutralising completely the effects of demand shocks. However the response to shocks affecting the commodity mark-ups is different. The natural interest rate reacts to fluctuations in actual commodity prices, which includes the mark-up (an increase of commodity prices driven by mark-ups call for an increase in the natural interest rate), while the efficient interest rate does not respond to changes in the mark-up.

3.3 Analysing policy rules

In log-linear terms, a Taylor rule that would implement closely the natural and efficient outcomes has the following form:

$$r_t = r_t^j + \phi_{core}\pi_{Y,t} + \phi_y \widehat{y}_t^j \quad \text{for } j = \{n, e\} \tag{3.10}$$

where the relative weights to core inflation and both definitions of the output gap determine the balance between core inflation and output gap stabilisation.

We analyse the macroeconomic dynamics under this especification of the policy rule for both the natural and the efficient benchmarks, taking into account preference demand and commodity supply shocks. The size of the shocks are standardised such that both generate a 1% increase in the commodity price under a benchmark scenario[7] and the coefficients of the policy rule have been calibrated as $\phi_{core} = 1.5$ and $\phi_y = 0.5$. For most structural parameters we have chosen a calibration that is in line with those used in the standard literature (Table 1). In the case of the parameters associated to the commodity market, because of the lack of a benchmark, we have chosen some conservative values. For example, for the share of commodities in consumption and production we have used a value of 0.05 for each share, and for size of competitive commodity production relative to GDP we used a value of 10%.

As shown in Graph 1, the policy response to demand shocks coincides under both types of benchmarks, such that an increase in the policy rate (panel 3) fully leans against the wind. Neutralising this way all the effects of demand shocks on macroeconomic variables (panels a and d), including those associated to the commodity market such as the commodity price and commodity demand (panels b and c).

In the case of commodity supply shocks the responses differ under both types of policy benchmarks (Graph 2). A negative commodity supply shock increases in both cases the commodity price (panel b) and reduces the supply of the competitive exporters (panel c), which partially offsets the increase in price via a reduction in the markup (panel b). Both the natural and efficient output benchmarks fall, but the drop in the efficient output is larger (panel d). Under the natural policy, both core inflation and the natural output gap are completely stabilised (blue lines in panels a and e), while under the efficient policy there is a trade-off: the efficient output gap becomes positive and core inflation becomes negative (red lines in panels a and e). Note that the optimal efficient policy corresponds to a partial offset of the effects of

[7] More precisely, the size of the shocks are calibrated such they generate a 1% increase in the commodity price when the monetary authority follows a standard Taylor rule of the form: $r_t = \phi_\pi \pi_{y,t} + \phi_y y_t$, where y_t corresponds to deviations from the steady state. The corresponding standard deviations of the shocks are 5.03 and 1.44 for preference demand and commodity supply shocks, respectively.

commodity prices on headline inflation by a reduction in core inflation (panel a red lines), and a larger decrease in output in comparison to the core inflation stabilisation case (panel d).

We also analyse the performance of the policy rule (3.10) in terms of welfare from the perspective of the representative household of the commodity importing country, for different calibrations of that rule. For completeness, we also compare it with two alternative policy rules:

$$
\begin{aligned}
r_t &= r_t^j + \phi_{core}\pi_{Y,t} + \phi_y \widehat{y}_t^j + \phi_{com}\Delta q_t && (3.11)\\
r_t &= r_t^j + \phi_{head}\pi_t + \phi_y \widehat{y}_t^j \qquad \text{for} \quad j = \{n, e\} && (3.12)
\end{aligned}
$$

Under specification (3.11) the monetary authority reacts also to commodity prices in addition to the variables in (3.10), while under policy (3.12) it reacts to headline inflation instead of core inflation. Note that both specifications are equivalente when $\phi_{com} = \gamma/(1-\gamma)$, which is the factor by which commodity prices affect headline inflation (equation 3.3).

In Table 2 we present the expected welfare cost in terms of steady state consumption from commodity supply shocks[8] and the combination of parameters that minimises such welfare loss. We do not present those corresponding to demand shocks because they are the same under all of these policy specifications. The first column corresponds to the efficient specification. Column 2 and 3 are for two calibrations of rule (3.11) such $\phi_{com} = \{0.05, 0.1\}$ and the last column is for rule (3.12). According to this, the optimal calibration of the policy rule (3.10) is such $\phi_y = 0$, such there is no response to the efficient output gap. This type of policy, has some positive but small quantitative welfare gains in comparison to the natural policy (not shown), which implies core inflation stabilisation for all calibrations. Note that the optimal Taylor rule (3.10) generates lower welfare costs than the policy rules that respond directly to commodity prices or headline inflation. This is because this rule (3.10) already takes into account the effects of commodity prices from an optimising welfare perspective from the policy benchmarks.

4 Policy misperceptions and monetary policy spillovers

In this section we analyse the consequences of which the central bank in the commodity importing country does not identify correctly the source of the shocks driving the fluctuations in commodity prices. As seen in the previous section, the optimal response to demand shocks is to fully neutralise its effects on output and inflation, such that commodity prices should not fluctuate. On the other hand, the optimal response to a negative commodity supply shock is to partially offset the effect on headline inflation by a reduction in both core inflation and output.

[8] The unconditional expected welfare (EW_t) is calculated using the second order solution of the model, where $W_t = U(C_t, L_t, g_t) + \beta E_t(W_{t+1})$ and the welfare cost in terms of steady state consumption is equivalent to $\{\exp[(1-\beta)(EW_t - W)] - 1\} \times 100$, given the logarithmic preferences of the respresentative household of the commodity importing country.

4.1 Signal-extraction problem

We assume the central bank in the importer country does not observe the fundamental shocks driving the fluctuations in commodity prices. That is, the central bank cannot disentangle if the fluctuations in the commodity prices are driven by commodity supply shocks or by commodity mark-up shocks. In log-linear terms, the central bank does not observe z_t and ψ_t that determines the commodity price:

$$q_t = -z_t + \psi_t = H'\xi_t$$

where $H' = \begin{bmatrix} -1 & 1 \end{bmatrix}$ and $\xi_t = \begin{bmatrix} z_t & \psi_t \end{bmatrix}'$. Also, we define the unconditional variance of ξ_t as P, where $P \equiv var(\xi_t) = \begin{bmatrix} \sigma_z^2 & \sigma_{z\psi} \\ \sigma_{z\psi} & \sigma_\psi^2 \end{bmatrix}$.

We further assume the central bank solves a signal-extraction problem by Kalman filtering to estimate the drivers of commodity prices, such:

$$E_t^{cb} \begin{bmatrix} z_t & \psi_t \end{bmatrix}' = Mq_t \tag{4.1}$$

where $M = PH\left[H'PH\right]^{-1}$ [9]. M is a weighted average of the variances and covariances of z_t and ψ_t, defined by:

$$M = \frac{x}{x^2 - 2\rho x + 1} \begin{bmatrix} \rho - x \\ \frac{1}{x} - \rho \end{bmatrix} \tag{4.2}$$

where $\rho = corr(z_t, \psi_t)$ and $x = \sigma_z/\sigma_\psi$.

It is worth to analise two extreme cases. In the first one (type A), when $x \to 0$, the volatility of the commodity market mark-up is very high relative to that of commodity market supply shocks. Then, in this case he central bank attributes all the fluctuations in commodity prices to mark-ups. That is:

$$\text{if } x \to 0, \qquad E_t^{cb} \begin{bmatrix} z_t & \psi_t \end{bmatrix}' \to \begin{bmatrix} 0 & q_t \end{bmatrix}'; \tag{4.3}$$

In the other extreme case (type B), when $x \to \infty$, all the fluctuations in commodity prices are attributed to supply shocks:

$$\text{if } x \to \infty, \qquad E_t^{cb} \begin{bmatrix} z_t & \psi_t \end{bmatrix}' \to \begin{bmatrix} -q_t & 0 \end{bmatrix}'. \tag{4.4}$$

In the general case (type C), the central bank attributes commodity price fluctuations to both components of the commodity price, taking into account their relative volatity and correlation as in equation (4.2).

Given the estimates of the drivers of commodity price fluctuations ($E_t^{cb}(z_t)$ and $E_t^{cb}(\psi_t)$), the central bank estimates the policy benchmarks ($E_t^{cb}(r_t^e)$ and $E_t^{cb}(\widehat{y}_t^e)$) to be used in the policy rule (3.10).

[9] See Hamilton (1994), chapter 13 for a derivation.

4.2 Policy misperceptions

As defined in the previous section, the optimal policy can be obtained with the following Taylor-rule:

$$r_t = r_t^{ef} + \phi_\pi \pi_{y,t} + \phi_y \widehat{y}_t^e$$

where ϕ_π and ϕ_y capture the relative weight between stabilising inflation and the welfare-relevant output gap. Given $E_t^{cb}(z_t)$ and $E_t^{cb}(\psi_t)$, the central bank estimate $E_t^{cb}(r_t^e)$ and $E_t^{cb}(\widehat{y}_t^e)$. If the central bank fails to correctly identify the shocks driving the fluctuations in the commodity price, it estimates r_t^e and $\widehat{y}_t^e$ with an error. That is:

$$\begin{aligned} r_t &= E_t^{cb}(r_t^e) + \phi_\pi \pi_{y,t} + \phi_y E_t^{cb}(\widehat{y}_t^e) \\ &= r_t^e + \phi_\pi \pi_{y,t} + \phi_y \widehat{y}_t^e + e_t \end{aligned} \tag{4.5}$$

where $e_t \equiv \left[E_t^{cb}(r_t^e) - r_t^e\right] + \phi_y \left[E_t^{cb}(\widehat{y}_t^e) - \widehat{y}_t^e\right]$ corresponds to a misperception error, which is an endogenous variable.

In Graph 3 we show the impulse responses to a commodity supply shock in the misperception case A. In this case, commodity price fluctuations are driven by a supply shock, but the central bank confuses it with a demand driven commodity mark-up shock. Consistent with that perception, the efficient interest rate is estimated as to have increased and the efficient level of output is estimated as constant (purple lines in panels f and d). In the overall, the error in the policy rate is positive (dotted blue line in panel f) which implies a more contractive monetary policy. As a consequence, core inflation (panel a) is lower than in the full information case (Graph 2), and the fall in output (panel d) is larger. Therefore, if the central banks fails to recognise that an increase in commodity prices is driven by external supply conditions, the consequences would be an excessive drop in both output and inflation. On the other hand, under the misperception case A, the response to demand shocks is similar to the full information case (Graph 1).

In graph 4 we show the impulse responses to demand shocks in the misperception case B, such that the fluctuations in commodity prices are attributed to supply shocks. As shown, the central bank estimates the efficient interest rate as unchanged and the the efficient level of output as if it has slightly decreased (purple lines in panels f and d). For these reasons, the policy rate (panel f) increases less than in the full information case (Graph 1), generating fluctuations in both output and inflation (panel a and d). Therefore, this policy misperception generates fluctuations in the commodity market that would have not happen if the central bank would have identified correctly its source.

In graph A1 and A2 in the appendix we show the general case, in which the central bank attributes commodity price fluctuations to both domestic demand and commodity supply factors. The general results from the previous exercises hold: in this case the central bank would respond excessively to commodity supply shocks, but less than adequate to demand shocks. In both cases, it would exacerbate fluctuations in commodity prices.

4.3 Monetary policy spillovers

To analyse the effects of monetary policy spillovers we plan extend the model assuming there is more than one importer country. That is, there are $n > 1$ importers countries indexed by i

such they face a similar signal-extraction problem as defined in this section. The Taylor rule that follows the central bank of each country has the following form

$$r_t^i = r_t^{ef,i} + \phi_\pi \pi_{y,t}^i + \phi_y \widehat{y}_t^{ef,i} + \mu_t^i$$

where the policy misspercetions errors are correlated between them $corr\left(\mu_t^i, \mu_t^j\right) \neq 0$.

5 Conclusions

[To be written]

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Table 1: Baseline Calibration		
Structural parameters		
Share of commodities in consumption basket	γ	0.05
Share of commodities in production function	α	0.05
Inv. Frish labour supply elasicity	v	0.5
Price elasticity of substitution	ε	7.66
Quarterly discount factor	β	0.99
Price adjustment probability	θ	0.75
Final goods' productivity in steady state	A	1
Commodities productivity in steady state	Z	1
Size of competitive commodity production relative to GDP	X/Y	10%

Table 2: Expected welfare lost for alternative policy rules
(as a % of steady state consumption)

$r_t =$	$r_t^e + \phi_{core}\pi_{Y,t} + \phi_y \widehat{y}_t^e$	$r_t^e + \phi_{core}\pi_{Y,t} + \phi_y \widehat{y}_t^e + \phi_{com}\Delta q_t$		$r_t^e + \phi_{head}\pi_t + \phi_y \widehat{y}_t^e$
ϕ_{core}	2.0	1.5	2.0	–
ϕ_{head}	–	–	–	1.3
ϕ_y	0.0	0.4	0.8	0.4
ϕ_{com}	–	0.05	0.10	–
Welfare lost	-1.231e-4	-1.236e-4	-1.292e-4	-1.251e-4

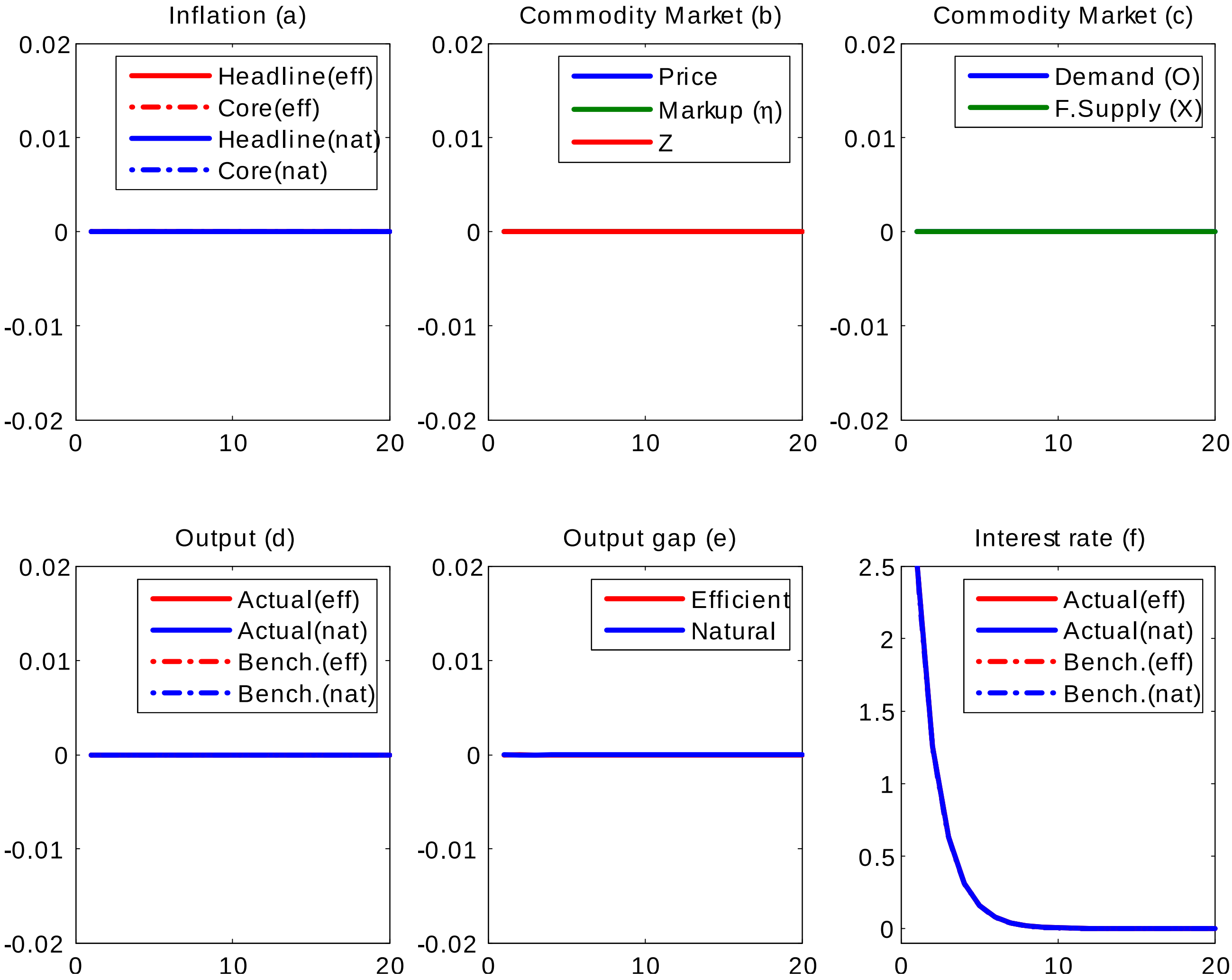

Figure .1: Response to a preference demand shock following the Taylor Rule: $r_t = r_t^j + \phi_{core}\pi_{Y,t} + \phi_y \widehat{y}_t^j$, under the natural ($j = n$) and the efficient benchmarks ($j = e$).

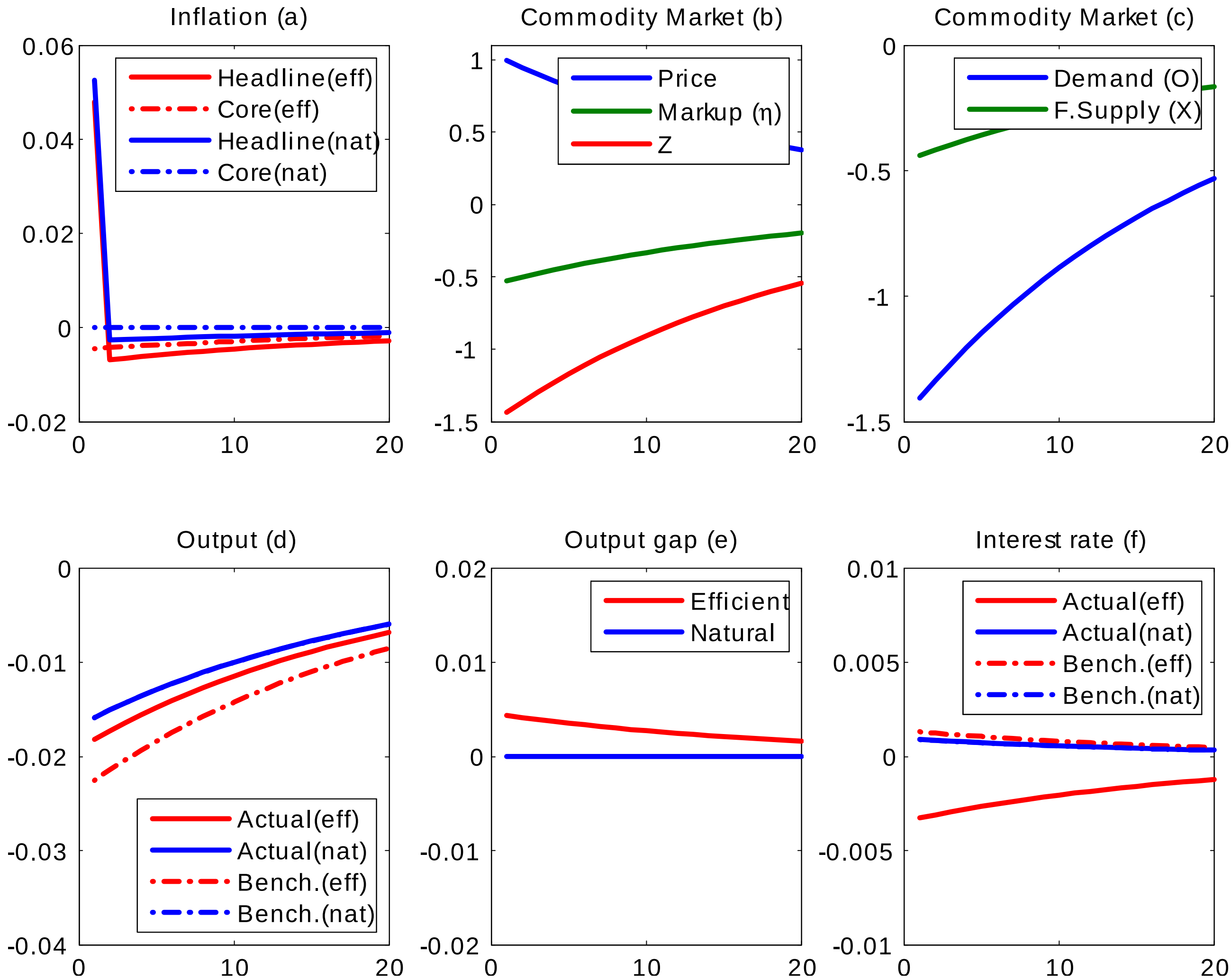

Figure .2: Response to a negative commodity supply shock following the Taylor Rule: $r_t = r_t^j + \phi_{core}\pi_{Y,t} + \phi_y \widehat{y}_t^j$, under the natural ($j = n$)and the efficient benchmarks ($j = e$).

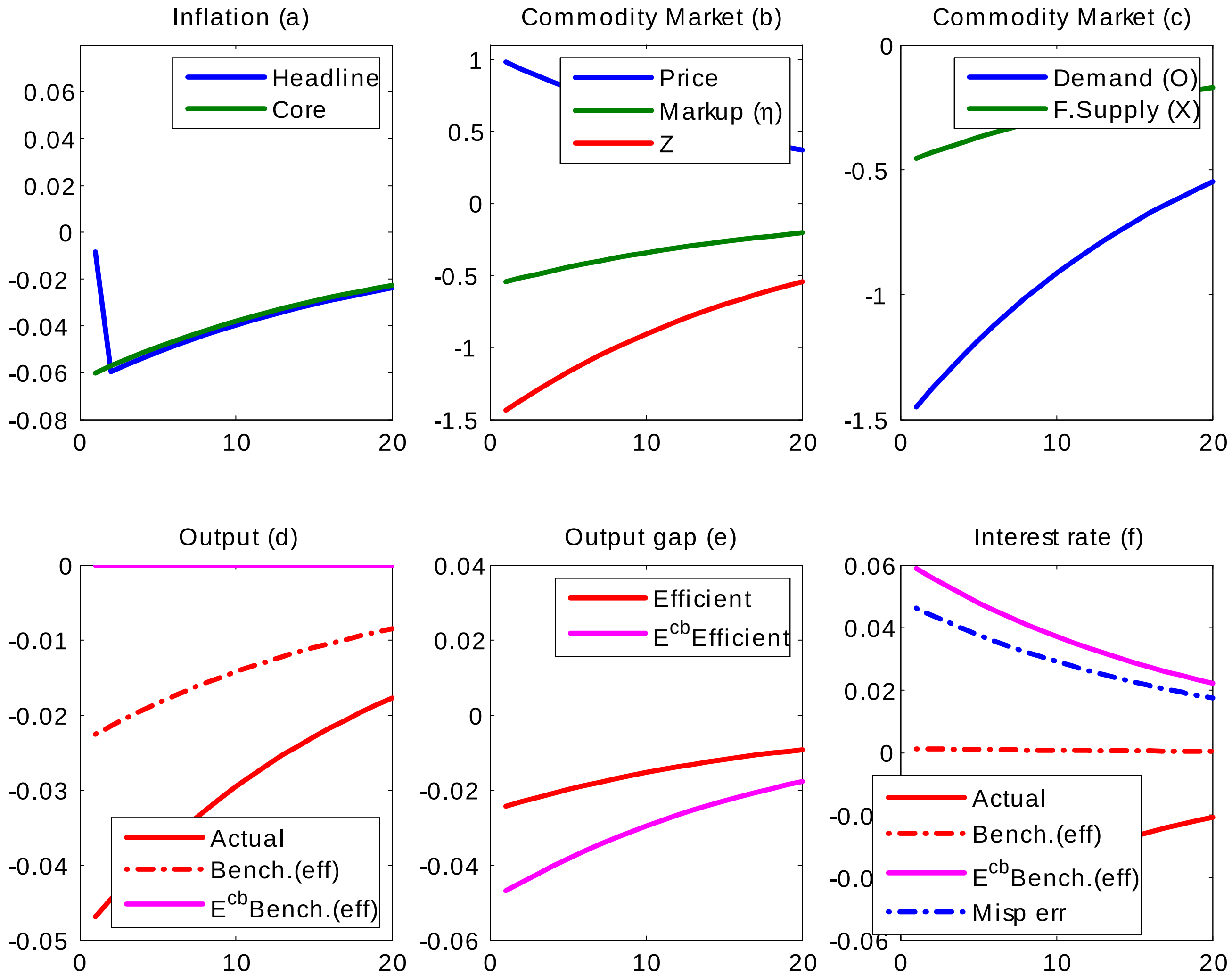

Figure .3: Impulse responses to a negative commodity supply shock when the central bank attributes all the fluctuations in commodity prices to markup changes (Misperception type A)

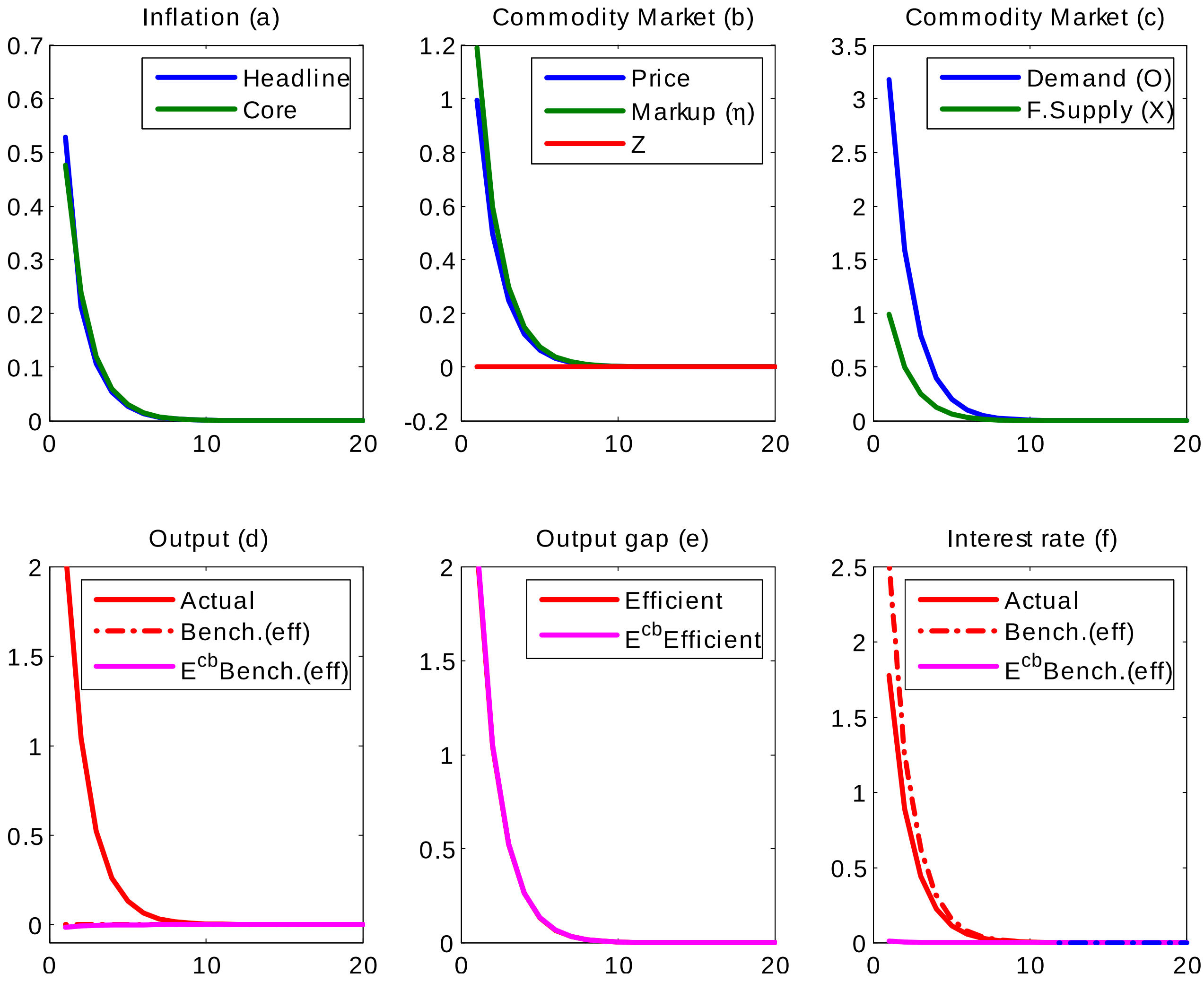

Figure .4: Impulse responses to a preference demand shock when the central bank attributes all the fluctuations in commodity prices to commodity supply shocks (Misperception type B)

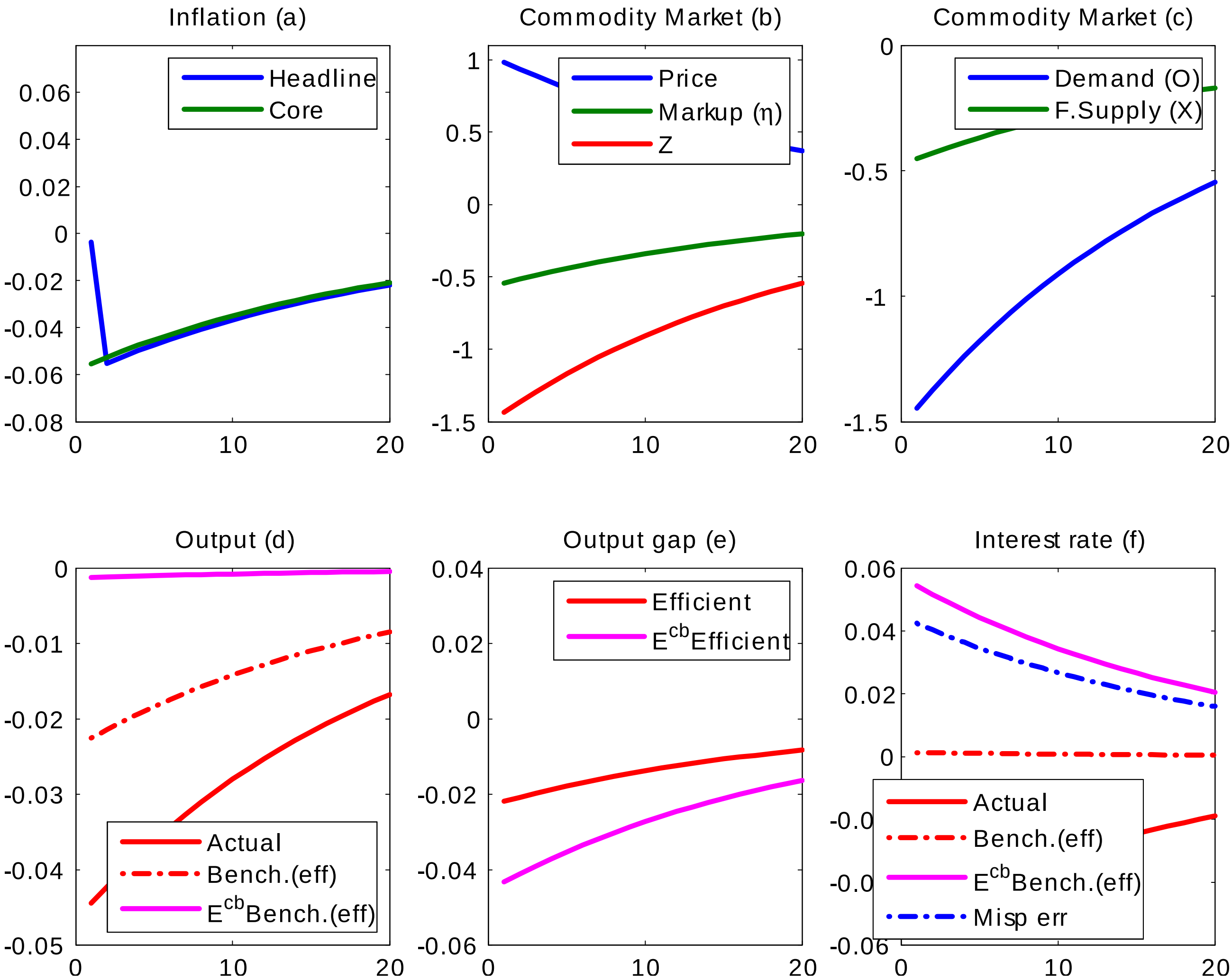

Figure A.1: Impulse responses to a negative commodity supply shock when the central bank attributes fluctuations in commodity prices to both markup changes and commodity supply shocks (Misperception type C).

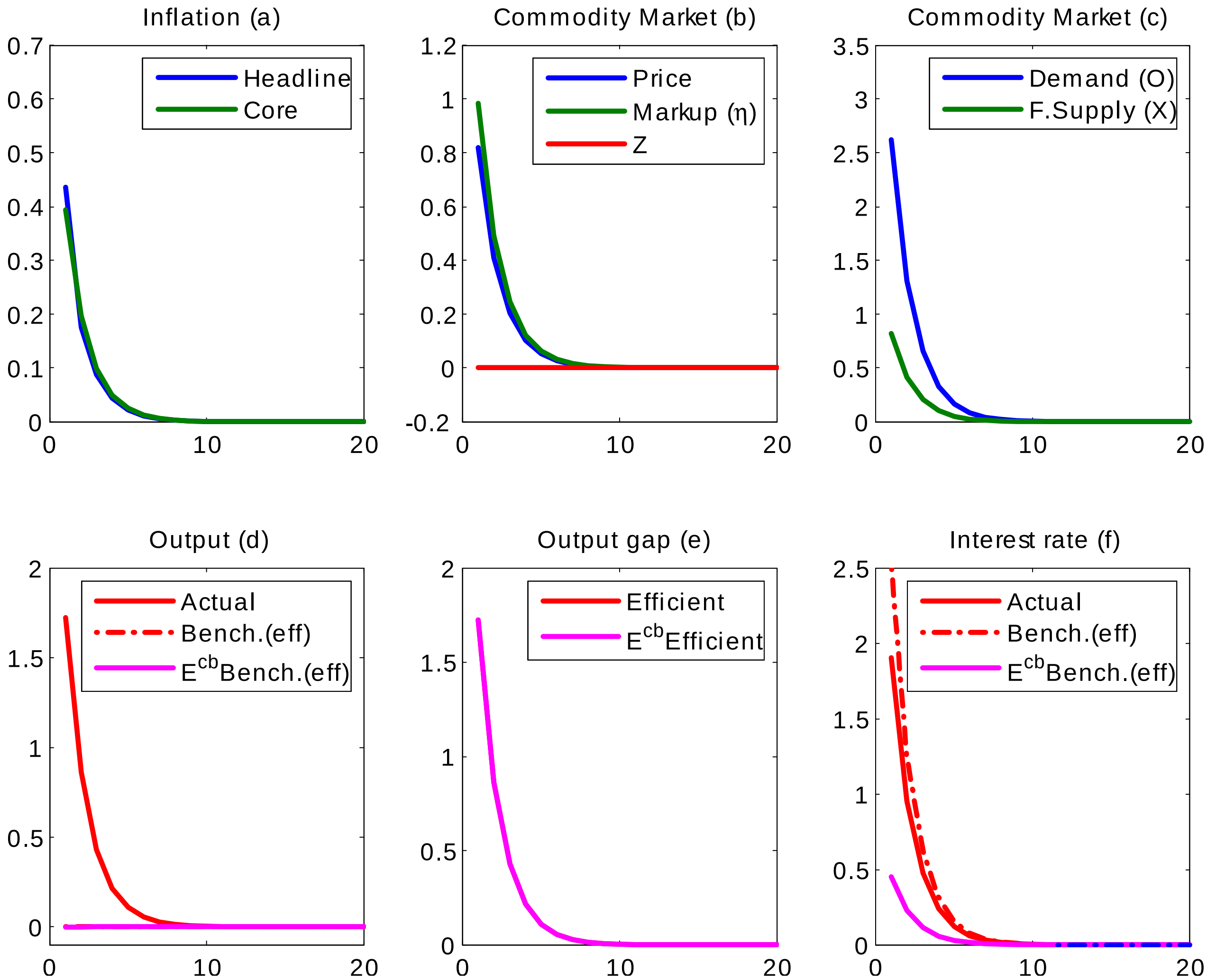

Figure A.2: Impulse responses to a preference demand shock when the central bank attributes fluctuations in commodity prices to both markup changes and commodity supply shocks (Misperception type C).

A Appendix: full set of equations

A.1 The commodity importing country

A.1.1 Aggregate demand

Aggregate demand equation (resource constraint)

$$\frac{P_{Yt}}{P_t} Y_t = C_t + Q_t O_{Yt} \tag{A-1}$$

IS equation

$$1 = \beta E_t \left[R_t \left(\frac{1}{\Pi_{t+1}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-1} \right] \tag{A-2}$$

Total consumption

$$C_t = (C_{Y,t})^{1-\gamma} (O_{C,t})^{\gamma} \tag{A-3}$$

Relative price final goods/total consumption goods ($P_{Y,t}/P_t$)

$$1 = \left(\frac{P_{Y,t}}{P_t} \right)^{1-\gamma} Q_t^{\gamma} \tag{A-4}$$

A.1.2 Aggregate supply

Total inflation:

$$\Pi_t = (\Pi_{Y,t})^{1-\gamma} \left(\frac{Q_t}{Q_{t-1}} \Pi_t \right)^{\gamma} \tag{A-5}$$

Core inflation Phillips Curve:

$$\begin{aligned} D_t &= Y_t (C_t)^{-1} + \theta \beta E_t \left[(\Pi_{Y,t+1})^{\epsilon-1} D_{t+1} \right], && \text{(A-6)} \\ N_t &= \mu Y_t (C_t)^{-1} MC_t + \theta \beta E_t \left[(\Pi_{Y,t+1})^{\epsilon} N_{t+1} \right], && \text{(A-7)} \\ \theta (\Pi_{Y,t})^{\varepsilon-1} &= 1 - (1-\theta) \left(\frac{N_t}{D_t} \right)^{1-\epsilon}, && \text{(A-8)} \end{aligned}$$

Price dispersion (only second order effects)

$$\Delta_t = (1-\theta) \left(\frac{N_t}{D_t} \right)^{-\epsilon} + \theta \Delta_{t-1} (\Pi_{Y,t})^{\varepsilon} \tag{A-9}$$

Marginal costs

$$MC_t = \left(\frac{W_t}{P_t} \right)^{1-\alpha} (Q_t)^{\alpha} / \left[A_t (1-\alpha)^{1-\alpha} \alpha^{\alpha} \right] \tag{A-10}$$

A.1.3 Labour market

Labour supply

$$\frac{W_t}{P_t} = C_t L_t^v \tag{A-11}$$

Labour demand

$$L_t = (1-\alpha)\,\frac{MC_t}{W_t/P_t} Y_t \Delta_t \tag{A-12}$$

A.1.4 Monetary policy

$$R_t = \overline{R}\,(\Pi_t)^{\phi_{head}}\,(\Pi_{Yt})^{\phi_{core}}\left(\frac{Q_t}{Q_{t-1}}\right)^{\phi_{comm}} \tag{A-13}$$

A.2 The commodity sector / countries

Total commodity demand to the dominant producer

$$O_t = O_{C,t} + O_{Y,t} - X_t \tag{A-14}$$

Commodity demand - consumption

$$O_{C,t} = \gamma \frac{1}{Q_t} C_t \tag{A-15}$$

Commodity demand - production

$$O_{Y,t} = \alpha \frac{MC_t}{Q_t} Y_t \Delta_t \tag{A-16}$$

Supply by competitive fringe

$$X_t = \Omega_t Q_t Z_t \tag{A-17}$$

Commodity supply (depends on the assumption of price determination in the commodity exporting country)

a) Perfect competition

$$Q_t^{PC} = Z_t^{-1} \tag{A-18a}$$

b) Imperfect competition

$$Q_t^{IC} = \Psi Z_t^{-1} \tag{A-18b}$$

$$\Psi_t = 1 + \frac{O_t}{2X_t} \tag{A-19b}$$

B Appendix: full set of linear equations

B.1 The commodity importing country

B.1.1 Aggregate demand

Aggregate demand equation (resource constraint)

$$\frac{P_Y}{P}\left(t_{y,t}+y_t\right)=\frac{C}{Y}\left(c_t\right)+Q\frac{O_Y}{Y}\left(q_t+o_{y,t}\right) \tag{B-1}$$

IS equation

$$c_t-E_tc_{t+1}=-\left(r_t-E_t\pi_{t+1}\right) \tag{B-2}$$

Total consumption

$$c_t=\left(1-\gamma\right)c_{y,t}+\gamma o_{c,t} \tag{B-3}$$

Relative price ($T_{y,t}=P_{Y,t}/P_t$)

$$0=\left(1-\gamma\right)t_{y,t}+\gamma q_t \tag{B-4}$$

B.1.2 Aggregate supply

Total inflation:

$$\pi_t=\left(1-\gamma\right)\pi_{y,t}+\gamma\left(q_t-q_{t-1}+\pi_t\right) \tag{B-5}$$

Core inflation Phillips Curve:

$$\pi_{y,t}=\kappa mc_t+\beta E_t\pi_{y,t+1} \tag{B-6}$$

Marginal costs

$$mc_t=\left(1-\alpha\right)wp_t+\alpha q_t-a_t \tag{B-7}$$

B.1.3 Labour market

Labour supply (making $WP_t=W_t/P_t$)

$$wp_t=c_t+vl_t \tag{B-8}$$

Labour demand

$$l_t=mc_t-wp_t+y_t \tag{B-9}$$

B.1.4 Monetary policy

$$r_t=\phi_{head}\pi_t+\phi_{core}\pi_{y,t}+\phi_{comm}\left(q_t-q_{t-1}\right) \tag{B-10}$$

B.2 The commodity sector / countries

Total commodity demand

$$\frac{O}{Y}o_t = \frac{O_C}{Y}o_{c,t} + \frac{O_Y}{Y}o_{y,t} - \frac{X}{Y}x_t \tag{B-11}$$

Commodity demand - consumption

$$o_{c,t} = c_t - q_t \tag{B-12}$$

Commodity demand - production

$$o_{y,t} = mc_t + y_t - q_t \tag{B-13}$$

Supply by competitive fringe

$$x_t = \omega_t + z_t + q_t \tag{B-14}$$

Commodity supply (depends on the assumption of price determination in the commodity exporting country)

a) Perfect competition

$$q_t^{PC} = -z_t \tag{A-15a}$$

b) Imperfect competition

$$\begin{aligned} q_t^{NI} &= -z_t + \psi_t && \text{(B-15b)} \\ \psi_t &= \left(\frac{\Psi - 1}{\Psi}\right)(o_t - x_t) && \text{(B-16b)} \end{aligned}$$

The exogenous variables are: z_t, ω_t , a_t and g_t.

C The steady state

The equations that determine the steady state of the equations of the commodity sector are the following:

$$\Psi = 1 + \frac{O/Y}{2X/Y} \tag{C-1}$$

$$Q = \Psi Z^{-1} \tag{C-2}$$

$$\frac{P_Y}{P} = Q^{-\gamma/(1-\gamma)} \tag{C-3}$$

$$\frac{X}{Y} = \frac{\Omega Z}{Y} Q \qquad \left(\frac{\Omega Z}{Y} \text{ given}\right) \tag{C-4}$$

$$\frac{O}{Y} = \left[\gamma \frac{P_Y}{P} + (1-\gamma)\frac{\alpha}{\mu}\right]\frac{1}{Q} - \frac{X}{Y} \tag{C-5}$$

This system of equations depend on the parameterisation of : $\alpha, \gamma, \frac{\Omega Z}{Y}, \mu, Z$.

Also, under zero steady state inflation, the steady state of the other equations that are needed to solve the model in log-linear form are:

$$\begin{aligned}
\Pi &= 1 \\
MC &= \frac{1}{\mu} \\
\Delta &= 1 \\
\frac{C}{Y} &= \frac{P_Y}{P} - \frac{\alpha}{\mu} \\
L &= \left[\frac{1-\alpha}{\mu} / \left(\frac{C}{Y}\right)\right]^{1/(1-v)} \\
Y &= \left[A\left(\frac{\alpha}{\mu}\frac{1}{Q}\right)^{\alpha}\right]^{1/(1-\alpha)} L
\end{aligned}$$

D The commodity exporter problem

Since there are no dynamics in the decisions, the problem of the commodity exporter under no internationalisation of action becomes:

$$\max \ln \left(Q_t O_t - O_t / Z_t\right) \tag{D-1}$$

subject to:

$$Q_t = f\left(O_t\right) \tag{D-2}$$

where (D-2) corresponds to equation (2.45) in the main text. The first order condition of this problem is:

$$Q_t^{NI} = Z_t^{-1} \frac{1}{1-\eta_t} \tag{D-3}$$

where $\eta_t \equiv -\frac{f'(O_t) O_t}{Q_t} = -\partial \ln Q_t / \partial \ln O_t$ is the elasticity of the demand for commodities in absolute value.

The net demand for commodities to the dominant producer can be written as:

$$O_t = \frac{1}{Q_t} D_t - Q_t E_t$$

where $D_t \equiv \left(\gamma C_t + \alpha M C_t Y_t \Delta_t\right)$ and $E_t \equiv \Omega_t Z_t$. Then, the inverse demand function is[10]

$$Q_t = \frac{1}{2} \frac{\sqrt{O^2 + 4 D_t E_t} - O}{E_t}$$

Given this, the elasticity of demand for commodities is:

$$\begin{aligned} \eta_t &= -\frac{f'\left(O_t\right) O_t}{Q_t} = \frac{O_t}{\sqrt{O_t^2 + 4 D_t E_t}} \\ &= \frac{O_t}{\sqrt{O_t^2 + 4\left(O_t + X_t\right) X_t}} = \frac{O_t}{O_t + 2 X_t} \end{aligned} \tag{D-4}$$

Then, the commodity market mark-up is:

$$\begin{aligned} \Psi_t &= \frac{1}{1-\eta_t} \\ &= 1 + \frac{O_t}{2 X_t} \end{aligned} \tag{D-5}$$

[10] The other solution is ruled out $Q_t = -\frac{1}{2} \frac{\sqrt{O^2 + 4AB} + O}{B}$ because it has negative values for the commodity price.

E Derivation of the benchmark output level

Replace the labour supply (2.6) in the aggregate labour demand (2.27) and solve for L_t:

$$L_t = \left[(1-\alpha) \frac{MC_t}{C_t} Y_t \Delta_t \right]^{1/(1+v)} \tag{E-1}$$

Replace C_t from the aggregate demand (2.34):

$$L_t = \left[(1-\alpha) \frac{MC_t}{\frac{P_{Yt}}{P_t} Y_t - Q_t O_{Yt}} Y_t \Delta_t \right]^{1/(1+v)} \tag{E-2}$$

Replace the commodity demand for output (2.28), making use of equation (2.10) to solve for Q_t and simplify:

$$L_t = \left[\frac{(1-\alpha) MC_t}{(Q_t)^{-\gamma/(1-\gamma)} - \alpha MC_t \Delta_t} \Delta_t \right]^{1/(1+v)} \tag{E-3}$$

This is total labour as a function of MC_t, Δ_t and Q_t.

Similarly, replace the commodity demand for ouptut (2.28) in aggregate output (2.29) and solve for Y_t:

$$Y_t = \left(\frac{A_t}{\Delta_t} \right)^{1/(1-\alpha)} L_t \left(\alpha \frac{MC_t}{Q_t} \Delta_t \right)^{\alpha/(1-\alpha)} \tag{E-4}$$

Replacing total labor from equation (E-3) gives the level of output in terms of the desired variables:

$$Y_t = \left(\frac{A_t}{\Delta_t} \right)^{1/(1-\alpha)} \left[\frac{(1-\alpha) MC_t}{(Q_t)^{-\gamma/(1-\gamma)} - \alpha MC_t \Delta_t} \Delta_t \right]^{1/(1+v)} \left(\alpha \frac{MC_t}{Q_t} \Delta_t \right)^{\alpha/(1-\alpha)} \tag{E-5}$$