Inventory Adjustments to Demand Shocks under Flexible Specifications

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Abstract

The relations among growth rates in GDP *and four aggregate demand components associated with inventory management are approximated by a neural* VAR *model with* t-*Student disturbances and an* ARCH *covariance matrix. The estimation sample corresponds to Peru's market-based growth experience (1993Q1-2010Q1). The main finding is that a positive shock to private demand growth will contemporaneously generate a more than proportional increase in production growth. This amplifier impact effect is consistent with the cycle of inventories and the average incidence of the inventory investment growth inside the production growth during the last four recessions.*

Keywords: time series models, neural networks, inventories, production smoothing, business fluctuations.

JEL classification: C32, C45, E22, E23, E32.

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1. INTRODUCTION

Since the 1940s the *inventory cycles* of Metzler (1941) have been recognized as a predominant characteristic of economic cycles (Blinder and Maccini, 1991). Their importance has been confirmed as lengthier expansions observed recently in the world economy came to an end with the United States (US) financial crisis (2007-2008) and the temporary collapse of international trade (see Alessandria et al., 2010), which also impacted the demand associated to commodity exports.

The sustained expansions seen since the start of the 1990s, particularly in the developed world, fostered growth in emerging economies due to greater commercial and financial openness. In general terms, this expansion was characterized by *1)* a decreasing and eventually low world inflation, an environment that had not been observed since the 1960s; and *2)* a reduction in the GDP growth volatility.

The literature on inventories have justifiably become relevant because they provide an explanation for the observed phenomenon of sustained and stable expansions, also known as the *Great Moderation*. According to this *explanatory history*, the continuous development of information technologies, communications, and sales forecasting techniques have fostered improvements in inventory management accompanied by a consequent reduction in the volatility of *inventory variation* (in other words, inventory investment or flow of inventory balances), which explains the reduction in the volatility of United States GDP and its corresponding growth rates (see, for instance, Kahn et al., 2002).

The prolonged economic expansion in US that started in 1991 was followed by a very short recession during the first few months of 2001, in which massive inventory liquidation was in contrast to the smooth movements observed previously, even before the prolonged expansion. For Kahn and McConnell (2002), this massive liquidation did not show that improvements in inventory management had been tenuous, but that firms had predicted falling sales long before they appeared, wich allow them to drastically reduce their inventories and thereby avoid excessive accumulation. Predicting the fall in sales allowed them to reduce their production in advance and then ration inventories according to demand, maintaining inventory-to-sales-ratios close to desired values.[1]

To characterize the stabilization observed in the US durable goods sector, Kahn (2008) points to two key facts: *1)* a significant reduction in the volatility of output growth and *2)* a more modest reduction in the volatility of sales growth. To characterize the stabilization of aggregate output in Australia, Simon (2001) also highlights two key facts: *1)* changes in the inventories cycle, and *2)* declines in *underlying output* volatility. By dismissing an increase in structural stability (the previously mentioned explanatory history), Simon (2001) explains the second key fact through a decline in the volatility of *productivity shocks* (supply shocks) that hit the economy, but leaves the source of such shocks as an open question.[2] In any case, the sectoral decomposition (by productive sectors) provides an explanation for the Great Moderation, which is complementary to that based on a decomposition of GDP growth by type of expenditure,[3] and both lines of work emphasize the unconditional variance of GDP.

1 The objectives of predicting sales and remaining close to the desired ratio imply that movements in inventories amplify business cycle fluctuations. Despite this, the *average* contribution of inventories to the volatility of GDP growth in the USA (its *average incidence*) is smaller. The model in the following section encompasses those contributions.

2 Later in this paper it will be seen that Simon's underlying output (2001) is actually an *aggregate demand excluding inventories*, meaning it is inappropriate to decompose it with an output function in order to estimate *productivity shocks*.

3 Eggers and Ioannides (2006) point to a decline in the importance within GDP of relatively more volatile sectors (agriculture and manufacturing) in favor of other less volatile ones (financial and services) as *the* explanation for the Great Moderation. Davis

The recent financial crisis in USA (2007-2008) affected demand associated with exports as part of the inventory cycle in the economic cycle, generating an unparalleled collapse and recovery of international trade. The literature has highlighted the role of private domestic demand and the inventories mechanism (Alessandria et al., 2010), as well as the private domestic demand of a country's main trading partner (Eaton et al., 2011), the latter being the most important determinant of external demand for exports.

In this context, although towards the start of 2010 it was too early to outline a general description of the turning point stemming from the US crisis in 2007-2008, the experience of Peru up until then might be illustrative of the inventory cycle in an emerging economy, despite having only a few business cycles, i.e., recorded under market conditions (Barrera, 2009). Moreover, economic relations between inventory growth during GDP growth shocks and three aggregate demand components (public domestic demand, private domestic demand and external demand for exports) stand out as being the least studied in Peru.

Table 1 quantifies the importance of inventory change as a percentage of GDP variation in the four recessions observed in Peru prior to that generated as a consequence of the US crisis in 2007-2008.[4]

The average of these coefficients is 230.4%, with a variation range of [100.9, 466.6]. As a reference, the average for the USA is 87%, with a variation range of [2, 232] according to the calculations made by Blinder and Maccini (1991) with the eight recessions recorded during 1948-1982. Firstly, this confirms that shifts in inventory investment have contributed by amplifying the recessive phases of the Peruvian economy since the start of the 1990s, especially the most recent one. Secondly,

and Kahn (2008) seek a more complete explanatory theory, with several interacting factors.

[4] The units employed are peak to trough changes in the *four-quarter average percentage variations* (percentage variations in four-quarter moving averages expressed in millions of 1994 nuevos soles).

Table 1

AVERAGE INCIDENCE OF INVENTORY CHANGES
Inventory investment and recessions since 1990

Reference variable: nonprimary GDP (peak-trough dates)	*Change in four-quarter average percentage variations (peak-trough)* Real GDP (1)	Inventory investment (2)	*Inventory investment to real GDP (2/1)*
Sample: 1992M12-2007M12[a]			
(1) 1995M7-1996M10	−2.4	−2.4	100.9
(2) 1997M12-1999M8	−1.7	−3.5	212.1
(3) 2000M8-2001M8	−1.8	−2.6	141.9
(4) 2003M3-2004M6	−0.7	−3.3	466.6
Average (1-4)	−1.6	−2.9	230.4
Memo: 2008Q2-2009Q2	−3.3	−12.2	373.9

[a] Four-quarter average percentage variations were the units employed to identify business cycles in Peru's economy using the Bry-Boschan approach (see Barrera, 2009).
Source: Author's calculations using data at levels from the Banco Central de Reserva del Perú.

and in contrast to the Great Moderation observed in US business cycles, Peru has seen a phenomenon of *demoderation*, at least since the third recession recorded.[5]

This paper aims to explain why the *demoderation* phenomenon takes place in Peru. To that end we quantitatively approximate the dynamic relations (potentially asymmetric) between inventory growth, GDP growth and three aggregate demand components (domestic public demand, domestic private demand and, especially, external demand for exports) during

[5] Note that the coefficients for USA are calculated using peak to trough flows in billions of 1982 dollars, making them indirectly comparable with coefficients for Peru.

Peru's market-based growth experience between the first quarter of 1993 and the first quarter of 2010 (1993Q1-2010Q1).

The second section describes the data that will be used to obtain empirical results. These data allow us to outline what we tentatively and temporarily call the *stylized facts* regarding the use of inventories. In principle, inventories serve to buffer the effect of demand shocks on manufacturing operations, although they can also be used for *other objectives* that would explain the demoderation phenomenon in Peru. The third section presents a conceptual framework with respect to production and inventory decisions to provide a qualitative explanation for the demoderation phenomenon. The fourth section proposes a flexible nonstructural model to approximate the dynamic asymmetric relations among GDP and inventories and three aggregate demand sources, as well as a structural model to decompose the covariance matrices of the last period in the sample (final period T= 2010Q1). The fifth section describes the results in terms of conditional covariance and impulse responses in an attempt to provide an explanation for the demoderation phenomenon in Peru. The sixth section gives the conclusions.

2. DATA AND STYLIZED FACTS: AGGREGATE FLOW OF INVENTORIES IN PERU

Data used in this study are taken from the Banco Central de Reserva del Perú and are available on its website under the title "Economic Statistics" at the following links: All Sets, Economic Activity and GDP expenditure, at <https://estadisticas.bcrp.gob.pe/estadisticas/series/trimestrales/pbi-gasto>. Figures are originally expressed in real 2007 soles.

2.1 Aggregate Data of Inventory, Production, and Demand

Aggregate production and inventories of firms in an economy will obviously respond to different types of demand shocks.

Thus, aggregate demand excluding inventory investment (*AgDem*) can be decomposed into:

1) real exports, goods and nonfinancial services (*XDem*);

2) public sector: real consumption and investment, goods and nonfinancial services (*PuDem*); and

3) private sector: real consumption and investment, goods and nonfinancial services (*PrDem*).

Figure 1 illustrates the *four-quarter average percentage variations* of those three components, and this *data transformation* will be used throughout the study.[6] To represent the scenario it was not sufficient to use the variance of the aggregate *AgDem:* fluctuations in *PuDem* were aimed at offsetting those in *PrDem* on several occasions (anticyclical policies) since the start of the 1990s (with a weak quantitative impact though), and since 1996 aimed to offset short term fluctuation in *XDem* (partially and at their discretion). Only since 2001, as comprehensive financial *constraints* on the public sector imposed during the economic stabilization were lifted, the frequency of these more focused countercyclical policies increased. These constraints consisted of continuous fiscal efforts to build public revenues to enable more effective medium-term anticyclical policies, which fostered a larger quantitative impact of fluctuations in *PuDem* during the sharp fluctuation in *XDem* caused by the crisis in the USA in 2007-2008.

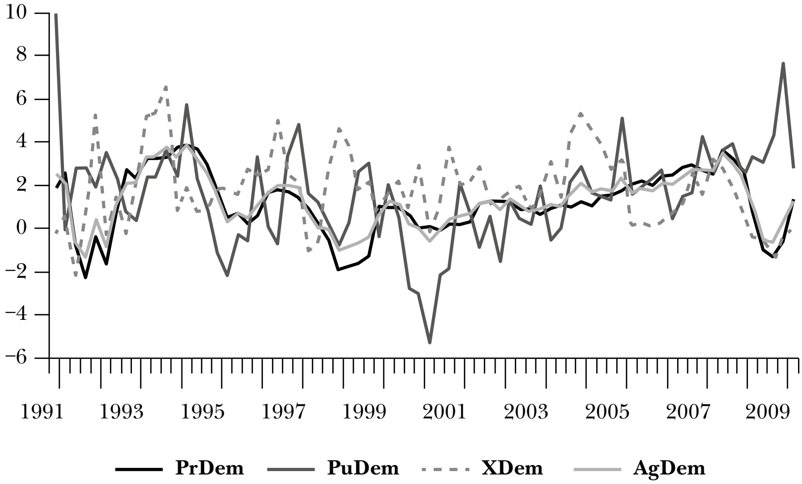
It is also important to consider two other endogenous variables: *1)* real calibrated balance of inventories (*BInv*), and *2)* real gross domestic product (GDP).

[6] One reason for not following the rules established in the literature for real cycles is due to the fact that data at levels contains a significant measurement error component, while these variations have a very low signal-noise ratio. Thus, Section 3 only provides a qualitative explanation that allows for structurally interpreting the empirical results in Section 5.

Figure 1

AGGREGATE DEMAND AND COMPONENTS

Four-quarter average percentage variations,
1991Q1-2010Q1



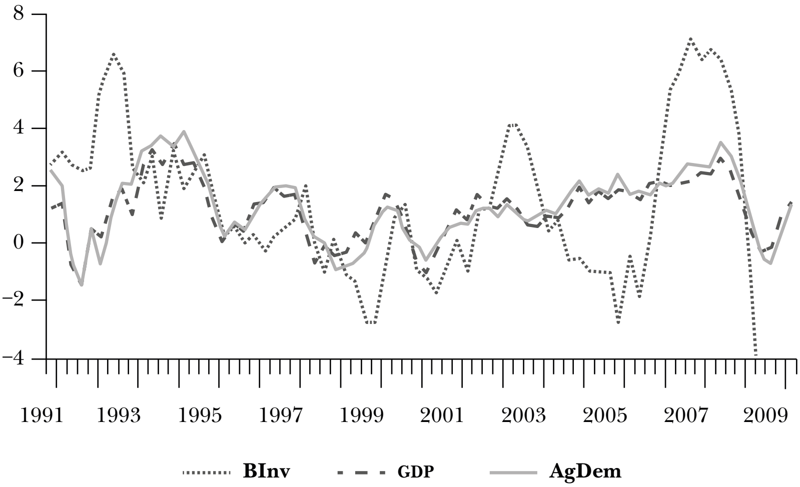
Source: author's calculations, using data at levels from the Banco Central de Reserva del Perú.

Figure 2 shows the same type of variations for these two variables, together with those of *AgDem*. It can be seen that *AgDem* and GDP grow at very similar rates. Meanwhile, the growth of *BInv* remained relatively close to aggregate demand since the reversal of the period of inventory overaccumulation at the end of 1994[7] and until the end of 1998. Later, three overaccumulations of growing magnitude are observed, the first ending

[7] This episode of overaccumulation reflected, at first, the recovery of production recorded by the success of the stabilization program (inflation decreased drastically, although still at double-digit levels), as well as the optimistic outlook for the economy before the end of the internal war, in the second half of 1992.

Figure 2

INVENTORY STOCK, PRODUCTION AND AGGREGATE DEMAND
Four-quarter average percentage variations
1991Q1-2010Q1



Source: author's calculations, using data at levels from the Banco Central de Reserva del Perú.

at the peak of 2000Q2; the second at that of 2003Q2; and the third at that of 2008Q1.[8]

Although only aggregate data is available for inventory investment, *DInv*, the increasing amplitude of *BInv* growth cycles might be explained by the growing participation of goods-in-process inventories in the whole *DInv*, particularly in traditional export sectors.

2.2 Stylized Facts for Aggregate Inventory Flows

The stylized facts on the relation between inventory investment, sales and production in Peru are presented similarly to

[8] The third reversal reaches negative rates of around 12% between 2009Q4 and 2010Q1.

in the literature on US inventories. We attempt to explain two stylized facts: *1)* why is production more or less volatile than sales? and *2)* why are inventory investment and sales not negatively correlated?[9]

One approach to these stylized facts comes from the unconditional sample moments of *four-quarter average percentage variations*, with quarterly frequency, of different GDP components by type of expenditure (one subaggregate of which is aggregate demand excluding inventory investment, *AgDem*). Table 2 presents the mean and standard deviation of these changes, as well as their correlations with inventory investment variation (*DInv*) and calibrated inventory stocks (*BInv*)[10] for two sample subperiods: before and during the period following the international financial crisis arising from the US crisis in 2007-2008.

In terms of standard deviations, production in Peru is *less* variable than sales (demand) for all *AgDem* components except private consumption (in both subperiods). Might there be incentives to use inventories as a buffer against positive demand shocks and maintain smooth production growth?

Given the extreme values in the means and standard deviations of *DInv* variation, the changes of calibrated inventory stocks, *BInv*, is a more stable indicator. This is confirmed in its correlations with the variance of all expenditure components.

Correlations with *DInv* variation reveal that variance in inventory investment and sales (demand) are positively correlated

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9 A third stylized fact emerges from recent improvements in the quality of inventory investment statistics for developed countries: The most volatile components of inventory investment are not finished goods inventories of the manufacturing sector, but rather its commodity inventories, as well as retail trade inventories (see Blinder and Maccini, 1991).

10 High levels of volatility in inventory investment growth rates in Table 2 justify such calibration (see Annex A) and explain emphasis on the relations between growth rates of aggregate production, aggregate demand and a *calibrated* sequence of inventory stocks.

Table 2

GDP BY TYPE OF EXPENDITURE (GOODS AND SERVICES)

Four-quarter average percentage variations

	Mean (M)				*Standard deviation (S)*				*Corr with D(Inv) (R1)*				*Corr with B(Inv) (R2)*			
	1995-2007		*2008-2010*		*1995-2007*		*2008-2010*		*1995-2007*		*2008-2010*		*1995-2007*		*2008-2010*	
	m()	*%m(GDP)*	*m()*	*%m(GDP)*	*s()*	*%s(GDP)*	*s()*	*%s(GDP)*	*r()*	*%r(GDP)*	*r()*	*%r(GDP)*	*r()*	*%r(GDP)*	*r()*	*%r(GDP)*
I. Global demand (1+2)	5.0	108.2	6.1	96.7	4.3	124.2	6.4	156.6	0.11	120.4	−0.02	−124.5	0.53	111.5	0.97	100.9
Ib. Global demand (1b+2)	5.0	107.3	7.3	117.1	4.1	118.4	5.3	128.7	0.11	120.8	−0.11	−613.8	0.50	104.5	0.98	101.9
1. Domestic demand with *D(Inv)*	4.5	97.5	6.4	102.3	4.9	144.3	6.6	161.6	0.14	154.9	−0.02	−103.9	0.58	121.5	0.97	100.8
1b. Domestic demand without *D(Inv)*	4.5	96.2	7.9	126.8	4.7	136.0	5.3	128.8	0.14	157.9	−0.13	−686.9	0.55	116.0	0.98	101.6
a. Private consumption	4.1	87.5	6.2	99.4	3.0	88.3	2.8	69.3	0.08	94.4	−0.15	−836.6	0.58	121.8	0.99	102.7
b. Public consumption	4.9	104.6	7.7	123.1	5.0	146.5	5.4	130.8	0.13	147.8	0.27	1,437.7	0.11	23.6	−0.95	−98.8
c. Gross domestic investment	6.3	136.2	7.6	121.0	13.0	380.1	20.5	500.8	0.17	186.9	0.00	20.7	0.57	120.6	0.97	100.3
Gross fixed investment (GFI)	6.0	129.5	13.8	221.0	11.8	344.1	15.9	388.8	0.18	201.7	−0.12	−638.0	0.52	109.5	0.98	101.6

Table 2 (cont.)

	Mean (M)				*Standard deviation (S)*				*Corr with D(Inv) (R1)*				*Corr with S(Inv) (R2)*			
	1995-2007		*2008-2010*		*1995-2007*		*2008-2010*		*1995-2007*		*2008-2010*		*1995-2007*		*2008-2010*	
	m()	*%m(GDP)*	*m()*	*%m(GDP)*	*s()*	*%s(GDP)*	*s()*	*%s(GDP)*	*r()*	*%r(GDP)*	*r()*	*%r(GDP)*	*r()*	*%r(GDP)*	*r()*	*%r(GDP)*
– Private	7.3	156.4	10.5	166.9	13.4	391.6	17.7	431.6	0.16	183.8	−0.13	−678.1	0.54	113.2	0.98	102.1
– Public	1.6	35.4	32.5	519.5	11.6	338.5	9.2	225.1	0.15	173.4	−0.06	−340.0	0.15	31.0	0.77	79.5
Stocks variance *(D(Inv))*	−83.3	−1,792.2	0.8	12.9	583.3	17,036.7	319.7	7,804.7	1.00	1,122.8	1.00	5,408.0	0.16	34.5	−0.23	−23.5
Memo: inventory stocks *(S(Inv))*	2.7	57.7	6.1	96.9	7.6	220.6	24.4	595.5	0.16	184.0	−0.23	−1227.3	1.00	210.5	1.00	103.8
2. Exports	8.4	181.2	4.3	68.4	4.3	125.4	5.4	132.6	−0.13	−142.9	−0.05	−258.9	−0.32	−67.9	0.97	100.8
II. Global supply (3+4)	5.0	108.2	6.1	96.7	4.3	124.2	6.4	156.6	0.11	120.4	−0.02	−124.5	0.53	111.5	0.97	100.9
3. GDP	4.7	100.0	6.3	100.0	3.4	100.0	4.1	100.0	0.09	100.0	−0.02	−100.0	0.48	100.0	0.96	100.0
– Goods	5.0	107.7	5.2	83.7	3.8	110.5	5.1	124.0	0.03	34.1	−0.01	−58.0	0.37	77.2	0.96	99.9
– Services	4.5	96.0	6.8	108.9	3.4	98.8	3.6	86.9	0.12	136.0	−0.03	−136.1	0.52	109.5	0.96	100.1
4. Imports	7.4	159.6	5.6	89.5	10.3	299.7	17.0	415.8	0.13	145.3	−0.03	−163.1	0.55	116.4	0.98	101.4

Source: Author's calculations using data at levels from the Banco Central de Reserva del Perú.

in the period 1995-2007 for all components of *AgDem* except exports. In period 2008-2010, they are negatively correlated, except public consumption. The size of all the correlations with *DInv* variation is close to zero due to the large proportion of noise present in *DInv*.

Correlations with the variation of *BInv* are more informative: changes in calibrated inventory stocks and sales (demand) are positively correlated in period 1995-2007 for all components of *AgDem* except exports. They are also positively correlated in period 2008-2010 except for public consumption. The magnitudes of this second group of correlations take values far from zero due to a clearer sign in calibrated stocks *BInv* (a small proportion of noise).

Why are variations of *BInv* and that of demand not negatively correlated? Could there be additional incentives for accumulating inventories at a *higher* rate than the minimum needed to cover positive shocks in demand growth and thereby slow or stabilize production growth?

If firms' main incentive for holding inventories is to meet positive shocks in demand growth and thereby smooth the evolution of production in order to leverage complementary opportunities such as, for instance, low input prices, firms are said to be producing to build stocks. In this case, changes in *BInv* allow them to control supply in response to demand fluctuations. Nevertheless, successive periods with higher than expected demand growth rates lead to increased production to meet part of the unanticipated demand and even achieve additional growth in *BInv*. This extra incentive for larger *BInv* growth stems from a need to increase a nonfinancial asset that offsets higher short-term borrowing incurred to cover production when demand is growing just in case this demand growth reverts (successive periods with lower expectations regarding demand growth could have symmetric effects). Hence, *BInv* and production operate in a coordinated manner, but with different periods, where *BInv* can be more than just the main instrument for offsetting demand shocks over the short-term.

Finally, although the stylized facts favor these hypotheses, it is worth questioning their suitability. Should we consider the description of those *unconditional* moments as a correct description of the stylized facts for the relations between aggregate demand growth, on the one hand, and growth in *BInv* and GDP, on the other? According to the variance decomposition theorem, the conditional variance of an available data set is less than unconditional variance. A more general theory sets forth that conditional covariance is different to unconditional covariance (which is also valid for the correlations). Therefore, the unconditional moments can only provide a preliminary description. In this regard, this paper aims to determine whether the conditional moments of the data provide evidence for the presence of the demoderation phenomenon in Peru.

3. GENERAL THEORETICAL MODEL WITH HETEROSCEDASTICITY

Sensier (2003) presents a model that encompasses those of Blanchard (1983), Blinder (1986), Eichenbaum (1989), Kahn (1987), and Ramey (1991) based on the model of Callen et al. (1990) and Cuthbertson and Gasparro (1993). Being I_t a vector of M levels of inventories held by a representative firm by type of good k, for instance, if $M=3$, $k=1$ (finished goods); $k=2$ (work-in-progress) and $k=3$ (raw materials), denominated in units of some finished consumer good that serves as a numeral. Moreover, the vector of functions for its corresponding desired levels is defined as

$$I_t^* = I^*\left(\underbrace{S_t}_{+}, \underbrace{z_t^I}_{-}, \underbrace{h_t^S}_{+}, \underbrace{r_t^H}_{-}\right), \tag{1}$$

where S_t is the vector of sales in period t[11] of M types of goods (to the market and to the firm's transfer pricing area), z_t^I is a

[11] It is feasible to interpret this function for desired inventory levels in period t as dependent on sales in period t, whatever this level

vector of technological change factors in period t for inventory control procedures for M types of goods, r_t^H is the financial-tax benefit of holding inventories as an asset[12] in period t and h_t^S is a vector of M standard deviations in period t of the prediction error (one period ahead) of each component of vector of sales S_t, conditional to all data available up to current period t. The signs under each variable suggest the direction of dependence in comparative statics (Callen et al., 1990, and Cuthbertson and Gasparro, 1993). The costs or losses incurred for moving away from desired levels are defined as the function

2 $$C_t^A = C^A(I_t - I_t^*),$$

that has been named *accelerator* in the literature.[13] The physical cost of holding inventories, which includes renting warehouse space, maintaining a suitable environment for preserving the qualities of the goods (for instance, refrigeration), transport equipment upkeep and man-hours for operating it, among others, is defined as the vector of functions

3 $$C_t^m = C^m(I_t, \delta),$$

where δ is the vector with M rates of depreciation (maximum effective decrease allowed) of each good k held in the firm's inventories (for instance, δ^f is the component corresponding

may be (including a predicted or expected level and elaborated with data available in any previous period $t-s$, where $s > 0$). The literature has typically considered it as dependent on expected sales for period t (see, for instance, Sensier, 2003; Cuthbertson and Gasparro, 1993; Blinder and Maccini, 1991).

[12] See Sensier (1993). Callen et al. (1990) treat it as a unitary financial cost for holding inventories.

[13] For instance, the sum of quadratic terms corresponding to each good k, multiplying each one by a coefficient $b_k/2$.

to finished goods).[14] The cost of producing finished goods is defined as the function

4 $$C_t^P = C^P(v_t, P_t),$$

where v_t is the marginal cost term that varies over time[15] and P_t is the level of production.[16] To simplify, from now on we assume that a firm only holds inventories of finished goods $\left(I_t = I_t^f\right)$, meaning all the vectors mentioned previously in this section are scalar.

The *inventories restriction* establishes a relation between production, sales, and finished goods inventory flows

5 $$P_t = S_t + \Delta I_t^f,$$

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[14] In Blinder (1982, 1986a, 1986b) and Sensier (2003), C_t^m is a quadratic function in I_t^f without a constant and with coefficient $e_2/2$ for the quadratic term. Eichenbaum (1989) use a quadratic function, but with coefficient e_{1t} for the linear term (that varies over time). Here the physical holding costs depend on depreciation rates (that can vary over time).

[15] In Eichenbaum (1989) it is a stochastic shock to the marginal cost of producing P_t in order for the model to encompass the motive for production cost smoothing of Blanchard (1983) and West (1990), such as for instance a shock to relative factor prices. In general, it can be any variable that affects a firm's intertemporal production decisions, such as financial or liquidity position (Cuthbertson and Gasparro, 1993; Sensier, 2003) or a one step ahead sales forecast error (Sensier, 2003 also uses production forecasts in her estimates).

[16] In Blanchard (1983), Eichenbaum (1989), Sensier (2003) and West (1990), C_t^P is a quadratic function in P_t without a constant and with coefficients v_t for the linear terms and $a/2$ for the quadratic term. If a is positive, marginal production costs are rising and the model encompasses the production smoothing motive of Blinder (1986a); if a is negative, the model includes the case considered by Ramey (1991).

that is normally used to obtain total flows (billed and unbilled) of finished goods sales. The historic sequence of inventory flows can be used to obtain inventory stocks, for instance, of finished goods,

6
$$I_t^f = \left(1-\delta^f\right) I_{t-1}^f + \quad I_t^f ,$$

i.e., an equation of perpetual inventories where δ^f is the depreciation rate of finished goods inventories.

Under these assumptions, the firm maximizes the conditional expectation of the present value of real benefit at time t, Π_t, with respect to the decision variable sequence, $\left\{I_{t+j}^f\right\}_{j=0}^{\infty}$, given predetermined variables I_{t+j-1}^f and sequences of the best forecasts of $\left\{S_{t+j}, z_{t+j}^I, r_{t+j}, h_{t+j}^S, v_{t+j}\right\}_{j=0}^{\infty}$ for the whole period considered in the present value, $\left[t, t+1, \cdots, \infty\right)$.

7
$$E_t\left[\Pi_t\right] \equiv \Big[\Sigma_{J=0}^{\infty} \beta^j \Big\{S_{t+j} - C^A\left(I_{t+j}^f - I^*\left(S_{t+j}, z_{t+j}^I, r_{t+j}, h_{t+j}^S\right)\right) \\ -C^m\left(I_{t+j}^f, \delta^f\right) - C^P\left(c_{t+j}, S_{t+j} + \Delta I_{t+j}^f\right)\Big\}\Big],$$

where β is the discount factor and $E_t[.] \equiv E_t\left[. \mid \Omega_t\right]$ is the conditional expectation operator for full relevant data set Ω_t available for a firm at time t when it is going to determine the optimal sequence $\left\{\tilde{I}_{t+j}^f\right\}_{j=0}^{\infty}$. We have assumed that the sales income function is concave and that cost functions are all convex, meaning the first order condition (Euler equation) is the necessary and sufficient condition for an optimal.[17]

Eichenbaum (1989) solves this problem for a set of specific parameters where the first order condition gives a necessary

[17] This formula should include benefits stemming from all production and financial operations conducted by a firm, or at least those associated to different types of inventories. For in-

and sufficient condition. After appropriate algebraic manipulation, he obtains the condition for the optimal plan of inventory stocks $\left\{\tilde{I}^{f}_{t+j}\right\}_{j=0}^{\infty}$, according to which:

1) I_t^f depends positively on expected future sales $\left\{S^{f}_{t+j}\right\}_{j=0}^{\infty}$: Inventories are held for production smoothing;

2) I_t^f depends negatively on current sales S_t^f: Because marginal production costs are increasing, there is a margin above which firms would rather cover their sales with inventories than increase production;

3) I_t^f depends negatively on a current stochastic shock to marginal production costs v_t: When marginal production costs are high, firms would rather meet current sales with current inventories than increase production;

4) I_t^f depends positively on future shocks to marginal productions costs $\left\{v_{t+j}\right\}_{j=1}^{\infty}$: Firms would rather build up inventories with current production when current marginal production costs are low compared to future ones, and therefore meet future sales out of those stocks of inventories instead of future production; and

5) I_t^f depends negatively on the linear coefficients (present and future) of inventory holding costs, $\left\{e_{1t+j}\right\}_{j=0}^{\infty}$ (see note 14).

Formulating the problem of a representative firm assumes that the variables are stationary. Given that production and aggregate sales are not stationary, the problem must be rewritten

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stance, costs associated to the factors of production for work in progress separated from finished goods, net benefits resulting from production operations for work-in-progress as well as financial operations such as purchases-sales of raw materials (the *inventories restriction* would be modified accordingly). The simple specification in terms of real benefits avoids considering the possibility for accounting part of the financial-speculative activities the corporate sector may perform with different types of inventories it holds.

through an appropriate normalization or alternatively by using a two-step approach proposed by Callen et al. (1990): *1)* propose a linear cointegration relationship between the *nonstationary* level of inventories and the determinants of a desired level of inventories; and *2)* use the cointegration error sequence to minimize total costs $C_t^T = C_t^m + C_t^P$ for each period as a function of inventory stocks.

The theoretical framework provides a qualitative explanation for the relation between the level of inventories and its determinants, although as mentioned previously, we will only use average percentage variations $(var\%)$ in the following sections.[18]

4. PROPOSED VARNN-ARCH MODELS

A family of dynamic models are immune to heteroskedasticity problems and are appropriate for both the conceptual model of the previous section and most models used in macroeconomics, where the aim is for the conditional means to be correctly calculated despite the presence of outliers and high variance episodes (Hamilton, 2008).

4.1 Conditional Means

First, we describe the models to be estimated for the conditional means. The first model for those moments is a typical linear multivalued function of VAR(K, p) models,

8 $$y_t = A_0 + A_1 y_{t-1} + \ldots + A_p y_{t-p} + \varepsilon_t = A_0 + \Sigma_{j=1}^{p} A_j y_{t-j} + \varepsilon_t,$$
$$\varepsilon_t \mid \Omega_{t-1} \sim N\left(0, \Sigma_t\right),$$

[18] Another justification for this can be found in the properties of elasticities ε_i of a scalar function that depends on n variables, $z_t = z\left(x_t^1, \ldots, x_t^n\right)$, or $var\%z_t = \sum_{i=1}^{n}\left(var\%x_t^i\right)\varepsilon_i$. The property is applicable to any of the functions used under this theoretical framework (including Euler conditions).

where $y_t' \equiv \{y_{1t}, y_{2t}, \ldots, y_{Kt}\}$ and $\varepsilon_t' \equiv \{\varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{Kt}\}$ are vectors of K stationary variables, $\Omega_{t-1} \equiv \{y_{t-1}', y_{t-2}', \ldots, y_{t-p}'\}$ is the relevant data set and $\Sigma_t \equiv \left[\sigma_t^{ij}\right]$ is matrix $K \times K$ of conditional covariances of period t ($\sigma_t^{ij} = \sigma^{ij}$ for VAR(K,p) models).

The second group of nonlinear VAR models generalizes the model of Equation 8:

9
$$y_t = A_0 + g(\Omega_{t-1}) + \varepsilon_t \quad \varepsilon_t \mid \Omega_{t-1} \sim N(0, \Sigma_t),$$

where it is usual to postulate a specific nonlinear multivalued function $g(.)$, for instance, choosing (somewhat arbitrarily) the smooth transition function, VSTVAR, or the self-excited threshold function SETVAR (see Granger and Teräsvirta, 1993).

Instead of assuming a priori the knowledge of function $g(.)$, a hypothesis that is taken as a premise in modern macroeconomics, here we employ a more general assumption: the existence of unknown nonlinear patterns in the data. Hence, we propose using flexible dynamic models (neural networks) whose main property is precisely a high capacity for approximating those patterns in the data. In this context, we choose a network architecture named *multilayer perceptron* (MLP).[19] Its dynamic version (VARNN-perceptron or VARMLP) will be used to obtain an approximation (global) of the nonlinear multivalued $g(.)$, the one that best adjusts to nonlinear patterns in the data.[20] According to that architecture, this is made possible by combining a finite number of basic structured nonlinear H functions in a multilayer graph,

[19] See Dorffner (1996). This architecture of *artificial neural networks* (ANN) are used in temporal series (also known as *feedforward* ANN; see Kuan and Liu, 1995).

[20] A Taylor approximation requires a specific function and an approximation point.

$$g\left(\Omega_{t-1}\right) \approx \beta_0 + \Sigma_{i=1}^{H} \beta_i h_i\left(\Omega_{t-1}\right) = \Sigma_{i=1}^{H} \beta_i \Psi_i\left(\Delta_{0,i} + \Sigma_{j=1}^{p} \Delta_i\left(j\right) y_{t-j}\right), \tag{10}$$

where H units h_i are denoted *hidden units*, each one of which is a multivalued linear function Ψ_i whose components are bounded functions.[21]

4.2 Conditional Covariances

Second, we describe the family of models for the conditional covariance matrices of the model we will finally estimate. This is the family of multivariate ARCH models, of which the most well-known are VECH, BEKK and exponential. The VECH model is the most general,

$$vech\left(\Sigma_t\right) = c + \Sigma_{h=1}^{p} C_h vech\left(\varepsilon_{t-h}\varepsilon'_{t-h}\right) + \Sigma_{k=1}^{q} B_k vech\left(\Sigma_{t-k}\right), \tag{11}$$

where using a *vech* operator (that stacks elements above and below the square matrix diagonal) gives c as a vector of order $\left[K\left(K+1\right)/2\right]\times 1$ and $\{C_h\}$, $\{B_k\}$ are matrices of order $\left[K\left(K+1\right)/2\right]\times\left[K\left(K+1\right)/2\right]$. As mentioned in Ding and Engle (2001), their generality goes hand in hand with their reduced parsimony and the difficulty of imposing restrictions that ensure a sequence of positively defined matrices $\{\Sigma_t\}$ (except when imposing $\{C_h\}$ and $\{B_k\}$ diagonals).

The BEKK model is a restricted version of the VECH model that generates a sequence of positively defined $\{\Sigma_t\}$ matrices by imposing a quadratic parameter structure,

[21] Shachmurove (2002) mentions that a major advantage of ANNs is their ability to analyze complex patters quickly with a high degree of accuracy and without making assumptions about the distribution of the data. Among the disadvantages are the fact they tend to over-fit data and lack a standard structured method for choosing, developing, training and evaluating an ANN.

12 $$\Sigma_t = CC' + \Sigma_{h=1}^{p} D_h \left(\varepsilon_{t-h}\varepsilon'_{t-h}\right) D'_h + \Sigma_{k=1}^{q} E_k \Sigma_{t-k} E'_k,$$

where C, $\{D_h\}$ and $\{E_k\}$ are matrices $K\times K$ and only C is a lower triangle. Engle and Kroner (1995) provide the conditions by which a BEKK model encompasses all diagonal VECH models with a sequence of positively defined matrices $\{\Sigma_t\}$ and almost all VECH models with a set of positively defined matrices $\{\Sigma_t\}$. These conditions eliminate all redundant representations (that are observed as equivalent).

The possibility of asymmetries in conditional covariances has been taken into account through two strategies. The first imposes specific restrictions not necessarily substantiated by the data (for instance, those proposed in Ebrahim, 2000; see Annex B in Barrera, 2010) while the second, proposed by Kawakatsu (2006), uses specific unrestricted parameterization, which we will use to adapt the model for this study.

Kawakatsu (2006) proposes a generalization of the asymmetric model of Nelson (1991) to the multivariate case that manages to maintain the generality of the VECH representation through an innovative parametric structure that generates a sequence of positively defined $\{\Sigma_t\}$ matrices without the sensitive simplifications of Ebrahim (2000). Using a VECH representation, Kawakatsu (2006) proposes

13 $$\begin{aligned} vech\left(log(\Sigma_t)\right) - c_0 = \Sigma_{h=1}^{p} C_t^{*} \varepsilon_{t-h} + \Sigma_{h=1}^{p} C_t^{**} \left(\left|\varepsilon_{t-h}\right| - E\left\{\left|\varepsilon_{t-h}\right|\right\}\right) \\ + \Sigma_{k=1}^{q} B_k \left(vech\left(log(\Sigma_{t-k})\right) - c_0\right), \end{aligned}$$

where $log(\Sigma_t)$ is the *matrix logarithm* of Σ_t, $vech\left(log\left(\Sigma_t\right)\right)$ and $c_0 \equiv vech(C)$ are vectors $\left[K(K+1)/2\right]\times 1$, C is a symmetrical matrix $K\times K$ and matrices $\left\{C_h^{*}\right\}$, $\left\{C_h^{**}\right\}$ and $\{B_k\}$ have dimensions $\left[K(K+1)/2\right]\times K$, $\left[K(K+1)/2\right]\times K$ and

$\left[K(K+1)/2\right]\times\left[K(K+1)/2\right]$, respectively. Matrices $\left\{C_h^{**}\right\}$ capture the *leverage effects* in the conditional covariance process.

Using the matrix logarithm transformation of the covariance matrix (symmetrical) means it is not necessary for $log(\Sigma_t)$ to be positively defined (or to impose any condition). Applying the exponential matrix (inverse) operation to that transformed space gives a covariance matrix that is symmetric and therefore positively defined. This allows any dynamic to be specified for this matrix, always generating a positively defined sequence of $\{\Sigma_t\}$ matrices.

If T is the number of observations, where $y_t' \equiv \{y_{1t}, y_{2t}, \dots, y_{Kt}\}$ is the transposed vector of K variables and Θ is the column vector of all the parameters, the normal multivariate conditional density of $y_t \mid \Omega_{t-1}$ can be written as:

14 $$f\left(y_t \mid \Omega_{t-1};\Theta\right) = (2\pi)^{-\frac{K}{2}} |\Sigma_t|^{-\frac{1}{2}} exp\left(-\frac{1}{2}\left(\varepsilon_t'\Sigma_t^{-1}\varepsilon_t\right)\right);$$

and log-likelihood function $l_Q = \Sigma_{t=1}^{T} l_t$, is obtained, where $l_t \equiv log\left(y_t \mid \Omega_{t-1};\Theta\right)$. For comparison purposes, the contribution of observation t to this log-likelihood function is

15 $$l_t = -\frac{1}{2}\left(Klog(2\pi) + log\left(|\Sigma_t|\right) + \varepsilon_t'\Sigma_t^{-1}\varepsilon_t\right).$$

In the case of the exponential matrix model of Kawakatsu (2006), this expression can be written as

16 $$l_t = -\frac{1}{2}\left(Klog(2\pi) + logm\left(\left|e^{logm(\Sigma_t)}\right|\right) + \varepsilon_t'\left(e^{logm(\Sigma_t)}\right)\varepsilon_t\right).$$

Using the following exponential matrix and matrix logarithm properties:

1) For all square matrix A, $\left(e^{A}\right)^{-1} = e^{-A}$.

2) For all symmetrical matrix S, $logm\left(\left|e^{S}\right|\right)=traza(S)$.

We obtain

17 $$l_t=-\frac{1}{2}\left(Klog(2\pi)+traza\left(logm(\Sigma_t)\right)+\varepsilon_t'\left(e^{-logm(\Sigma_t)}\right)\varepsilon_t\right).$$

By adding the exponential matrix of Kawakatsu (2006) to the proposed nonstructural modelling, which includes a multivariate Student's *t* distribution, all the parameters are robust to the presence of atypical observations without imposing specific restrictions not necessarily substantiated by the data. This model is estimated for the case of Peru with 65 quarterly data for the period 1994Q1-2010Q1.[22] Al the variables are expressed as *four-quarter average percentage variations.*

Estimation of the dynamic flexible econometric model is feasible, despite computing restrictions, if the over-parameterization problem is addressed. The latter is common in neural network models and can reduce their usefulness for predictive purposes. Annex B describes the *maximum penalized likelihood* method for solving this problem and the associated reduced number of degrees of freedom.

4.3 A Contemporaneous Structure

We proposed a structural model for covariance matrix decomposition for the final period $t=T$ of the nonstructural VARNN-ARCH model estimated (although the following discussion is applicable to the covariance matrix of any period t). Using decomposition *AB*, matrix $(I-A)$ is triangular and matrix B is dimension diagonal $k=5$. Ordering of the structural model $y_t\equiv\left\{XDem_t, PuDem_t, PrDem_t, BInv_t, GDP_t\right\}$ should be taken into account for interpreting its coefficients: The most

[22] The possibility of including the period of high inflation and its subsequent stabilization was rejected due to considerable fluctuations in relative prices. With the lags in conditional means and lags in conditional covariances, the estimated sample of conditional covariances includes 41 observations (2000Q1-2010Q1).

exogeneous shocks correspond to those of the growth rates of $\{XDem_t, PuDem_t, PrDem_t\}$, in response to which follows the compensatory action of the shock to the growth rate of $\{BInv_t\}$ (according to prevailing incentives), all of which finally determines the shock in the growth rate of $\{GDP_t\}$.

Expected values or signs of coefficients a_{ij} in the matrix (I–A) come from the theoretical model described in Section 3. We postulate that there are contemporaneous relations among shocks to the three aggregate demand components: It is anticipated that $\{PuDem_t\}$ fulfills some type of compensatory function in response to shocks in $\{PrDem_t\}$ and $\{XDem_t\}$ (*inverse* relations reflected in *positive* coefficients immediately below the main diagonal of the submatrix (1:3,1:3) of (I–A); see Table 3). Moreover, shocks in all three components affect firms' inventory and production decisions. If a firm's only incentive for holding inventories was production smoothing, the contemporaneous relations between $\{BInv_t\}$ and the three aggregate demand components would be *inverse* and reflected in *positive* coefficients in the fourth row of (I–A). However, if there are additional incentives for increasing $\{BInv_t\}$, these relations might be *direct* (*negative* coefficients in said row). Furthermore, while production smoothing, $\{GDP_t\}$, would free it from demand shocks (coefficients in the fifth row would be null), *additional incentives* would generate *direct* relations between supply shocks[23] and all the rest (*negative* coefficients in that row).[24]

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[23] As mentioned in Section 3, production shocks encompass marginal costs shocks (for instance, in the relative prices of factors of production) and technology shocks (investments that improve capital assets), but also include shocks to production processes (problems of logistics such as, for instance, cuts in energy supplies for manufacturing or mining, or shortages in inputs such as water for agricultural production, etc.).

[24] Section 2 does not mention that growth of $\{AgDem_t\}$ is a weighted average of the growth of the first three components of the vector

Table 3

MATRIX (I−A)

Affect a structural shock in:	*Structural shocks of*				
	XDem	*PuDem*	*PrDem*	*BInv*	*GDP*
XDem	1	0	0	0	0
PuDem	a_{21}	1	0	0	0
PrDem	a_{31}	a_{32}	1	0	0
Binv	a_{41}	a_{42}	a_{43}	1	0
GDP	a_{51}	a_{52}	a_{53}	a_{54}	1

5. RESULTS

From an econometric and statistical standpoint, it is worth questioning the relevance of using such general assumptions, performing statistical tests to validate the need for them, either individually or jointly. The answer, however, should consider the need to nest simpler hypotheses within the proposed model, a consideration that has proved hard to find in the literature consulted on the penalized likelihood (see Annex B).

The results of the proposed general observation tool that imposes a minimum number of maintained assumptions (with the additional cost associated with their estimation) are shown below. Another significant product of this tool is the availability of conditional covariances estimates (conditional variances indicate periods of greater uncertainty for each variable in the model).

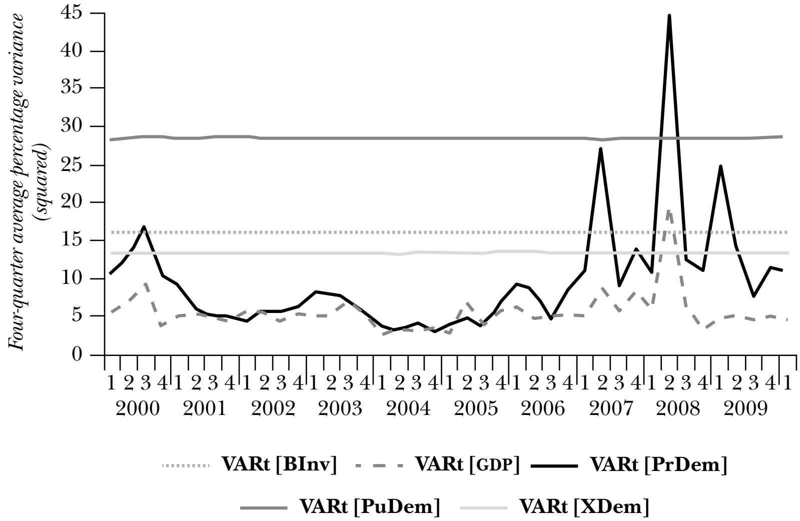
5.1 Nonstructural VARNN-ARCH Model

Figure 3 shows the conditional variance of four-quarter average percentage variations for each of the three aggregate

of endogenous variables.

Figure 3

CONDITIONAL VARIANCES OF THE AGGREGATE DEMAND COMPONENTS, INVENTORIES, AND PRODUCTION
2000Q1-2010Q1



demand components, inventories, and production (i.e., units are squared variations).

It can be seen that the conditional variances of *PrDem* and GDP change over time, while that of *BInv*, *PuDem,* and *XDem* appear as pseudo-constants due to the variation range of conditional variances that clearly change over time.[25]

Conditional variances of *PrDem* and GDP tend to rise contemporaneously, standing out the more recent jumps in uncertainty. Meanwhile, the sequence of conditional variances for GDP tends to be smaller than the sequence corresponding

[25] These two wide ranges of variation could reflect the need to separate *quanta* from relative prices inherent to original units (1994 nuevos soles) for including them in larger sized models (and difficult to estimate). In any case, all conditional moments of the model estimated are so with respect to the small number of included variables.

to *PrDem*, which reflects the existence of a degree of production stabilization with respect to *PrDem* that is attributable to inventory management and is more noteworthy in the event of jumps in the uncertainty of *PrDem*. As for pseudo constant conditional variances over time, that of the *PuDem* is greater than that of *BInv*, and that one is in turn larger than that of *XDem*. Given that these pseudo constants tend to be larger than the variances that change over time (*PrDem* and GDP), production stabilization is performed for each of those aggregate demand components.

With respect to the estimated conditional variance sequence for *AgDem*, which has been added to the previous figures, we calculated it based on the conditional covariances submatrix of its three components (*PrDem*, *PuDem*, and *XDem*).

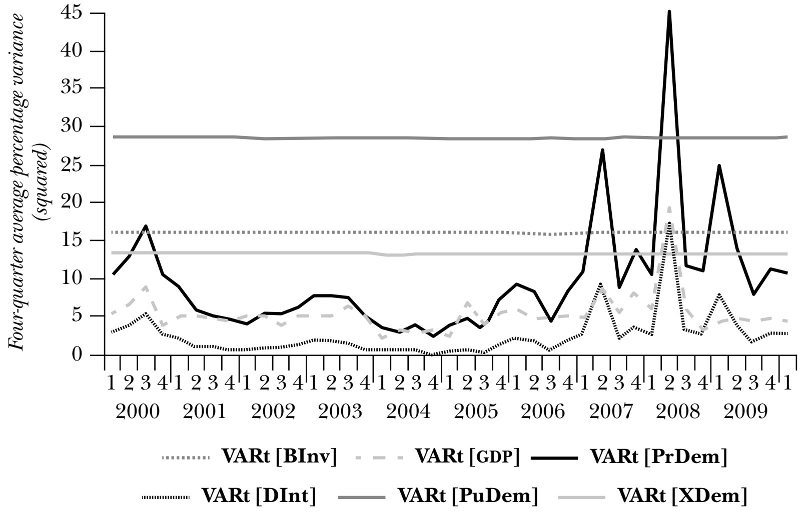
The conditional variance of *AgDem* confirms the possibility that motivated this study: That it is smaller than the conditional variance of GDP (except in one quarter subsequent to the recent period of maximum uncertainty) and with a relative magnitude of around one to four (during the period of low lower uncertainty). This result is in contrast to results obtained with unconditional variances (see Table 2), explained by the impact of conditional covariances among their three components.

To conclude, aggregate management of inventories leads to production stabilization through mechanisms that are reflected in the conditional covariances of variances in all three components of *AgDem* (*PrDem*, *PuDem,* and *XDem*). The evolution of all 15 different entries in the conditional covariance matrix (standardized) is presented in Annex C. Two out of the three covariances that intervene in calculating *AgDem* variance are negative, (*PrDem*, *XDem*) and (*PuDem*, *XDem*), which contributes to the sequence of the variance of *AgDem* being closer to abscissa (see Figure 4).

Covariances (*BInv*, *PuDem*) and (*BInv*, *XDem*) are negative, reflecting expected inverse relations when there are no other incentives for holding inventories except GDP smoothing. Covariance (*PrDem*, *BInv*) is the only positive one, reflecting the expected direct relations when there are additional incentives for *BInv* growth.

Figure 4

CONDITIONAL VARIANCES OF THE AGGREGATE DEMAND COMPONENTS, INVENTORIES, PRODUCTION, AND INVENTORY INVESTMENT
2000Q1-2010Q1



5.2 Structural VARNN-ARCH Model: Contemporaneous Structure

Table 4 displays the coefficients estimated for the matrices of *AB* decomposition of the conditional covariance matrix estimated for the last sample period (T=2010Q1). Note that items below the diagonal in $(I-A)$ have the opposite sign to those of the corresponding items of A, while items different to zero in matrix B (its diagonal) are shown as a column vector.

All the parameters estimated in the matrix $(I-A)$ for period T of the sample are statistically equal to zero, except the parameter that measures the *positive* impact of the *PrDem* structural shock on GDP (−1.207 in the table). Estimates in period T of the sample reveal that the contemporaneous relations between *BInv* and *AgDem* components are statistically equal to zero. Therefore, GDP growth smoothing is not the only incentive

Table 4

ESTIMATED CONTEMPORANEOUS RELATIONS
SpVARNN-ARCH with five variables

			I−A				
			1	*2*	*3*	*4*	*5*
		B	*XDem*	*PuDem*	*PrDem*	*BInv*	GDP
1	*XDem*	3.671	1.000				
		(1.985)					
2	*PuDem*	5.361	0.053	1.000			
		(1.679)	(0.123)				
3	*PrDem*	3.332	0.059	(0.047)	1.000		
		(1.112)	(0.074)	(0.118)			
4	*BInv*	4.040	0.044	0.172	(0.076)	1.000	
		(0.521)	(0.117)	(0.188)	(0.115)		
5	GDP	1.822	(0.013)	(0.002)	(1.207)	(0.008)	1.000
		(1.335)	(0.016)	(0.026)	(0.016)	(0.033)	

for increasing *BInv* in that period, meaning there are possibly *additional incentives* for it. The only parameter statistically different from zero is consistent with the presence of additional incentives, which according to the macroeconomic context of that period means that negative shocks in *PrDem* growth are reflected in decreases in production growth measured in GDP.

On the basis of these contemporaneous relations we obtain the response functions for any variable i after a 1% change in any variable j (*impulse response functions*), denoted as $FRI\left[j \to i\right]$. Impulse response functions (IRF) were calculated as the difference between two projections that are not based on a stationary state: a projection with the structural shock from period T, the last period of the sample, and a projection without this shock (see Koop et al., 1996).

The IRFs do not generally show asymmetries in the sign or magnitude of shocks, although the scale of contemporaneous impacts (responses in period T) dominate the scale of the rest of the sequence (responses in periods $T+h$, $h \neq 0$). For this reason, IRFs are presented in 2×2 subgraph matrices: IRFs in the first row include contemporaneous impacts, while those in the second row exclude them.[26]

5.3 Impulse Responses in the *PrDem*

Figure 5 displays IRFs for estimated increases in *BInv* and GDP after a shock of 1% in *PrDem* growth. This positive structural shock in *PrDem* causes GDP growth to increase at the time of impact, it falls soon after and then continues to decline slowly towards zero. Meanwhile, *BInv* growth increases upon impact, continues increasing very slowly and then falls 10 quarters ahead.

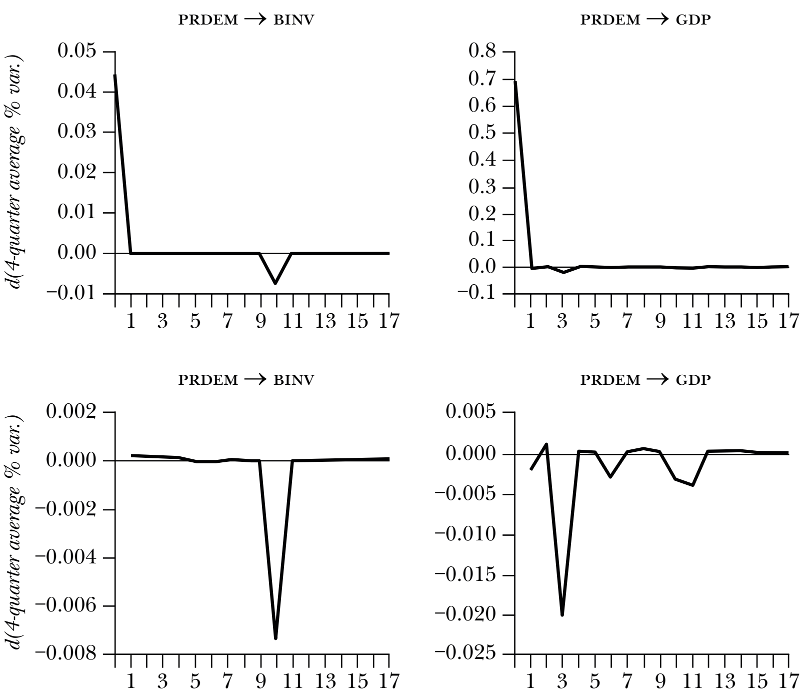
Considering the relative magnitudes, a positive structural shock in *PrDem* growth is initially absorbed by a sharp increase in the GDP growth and a slight increase in *BInv* growth (which is followed by a delayed smaller decrease 10 quarters ahead). This behavior is in disagreement with simple intuitive inventory management, but consistent with *additional incentives* for raising the growth of *BInv*, such as lags in the adjustment of the aggregate production process and induced price changes that maximize private profits (high current prices with respect to the marginal production costs of stocked goods, not necessarily finished goods).

The model estimated captures here the episodes where inventory investment amplifies the response of GDP to large negative demand shocks (during the recessive phases of Peru's

[26] The first row of graphs includes the value of the coefficient corresponding to the estimated contemporaneous impact in matrix A (Table 4), which is typically greater (in absolute value) than the contemporaneous impact in the corresponding IRF due to the way it was calculated.

Figure 5

IMPULSE-RESPONSE FUNCTIONS FOR ESTIMATED INCREASES IN BINV AND GDP AFTER A SHOCK OF 1% IN PRDEM GROWTH



economy since the start of the 1990s, particularly the most recent one), the *demoderation* phenomenon mentioned in Section 1.

Limitations to inventory statistics in Peru[27] make it necessary to postpone a strict comparison of a new hypothesis

[27] Barrera (2009) employs 12-month average percentage changes to date the phases of business cycles in Peru's economy with monthly periodicity. Using those units avoids problems for measuring real monthly levels, making the monthly dates for peaks

expounded in the literature that the recent international crises explain most of the recent fluctuations in the inventory cycle (especially in exportable primary products) and therefore in the activity of an increasingly globalized economy such as Peru's (see Alessandria et al., 2010).[28] This paper provides indirect evidence to support this hypothesis.

5.4 Impulse Responses in *PuDem*

Figure 6 displays IRFs for the estimated growth in *BInv* and GDP after a shock of 1% in *PuDem* growth. In response to a positive structural shock in *PuDem* growth, *BInv* growth falls upon impact and subsequently remains unchanged until it increases 10 quarters ahead. On the other hand, GDP growth increases upon impact, then rises very slightly and falls lightly after which it exhibits a series of small falls and rebounds with the zero line as a ceiling.

Given the relative magnitudes, an increase in *PuDem* growth is absorbed by a significant fall in *BInv* growth and a small increase in GDP growth. The tendering process associated with government expenditure, very different on aggregate from the private expenditure process, can explain this behavior more in line with intuitive inventory management, but opposite to that resulting from a shock in *PrDem* (of the same sign).

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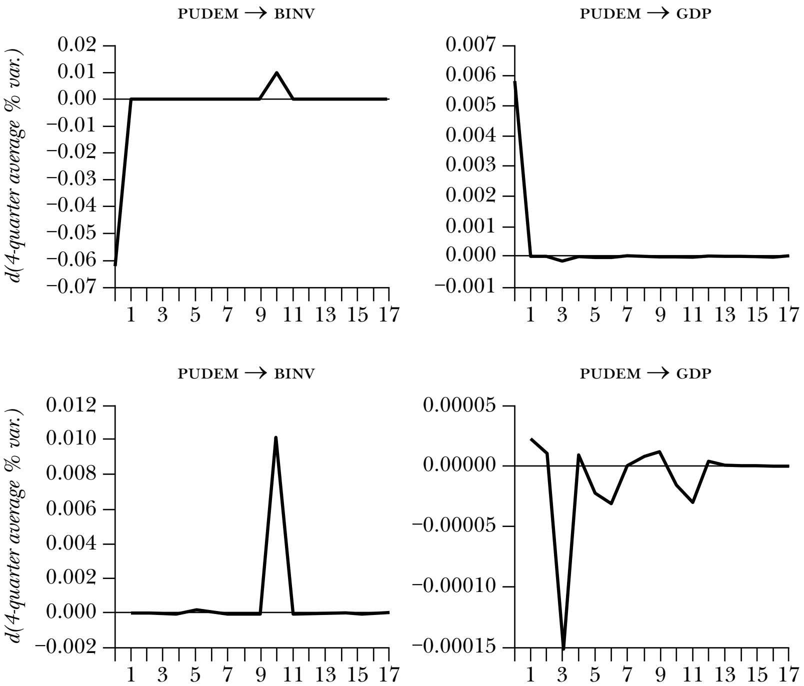
and troughs more robust. Given those dates, if the coefficients (inventory investment) / GDP of recessionary phases in Peru are calculated using real quarterly flows in millions of 1994 soles, only the coefficient corresponding to the recession between December 1997 and August 1999 (1997M12-1999M8) will be valid.

[28] Disaggregating inventory investment into its typical components (inputs, work-in-progress and finished goods) is not feasible with data for Peru, and even less so with its external trade components (exports, imports and nontradeable goods). The latter disaggregation is used by Alessandria et al. (2010) for USA.

Figure 6

IMPULSE-RESPONSE FUNCTIONS FOR ESTIMATED INCREASES IN BINV AND GDP AFTER A SHOCK OF 1% IN PUDEM GROWTH

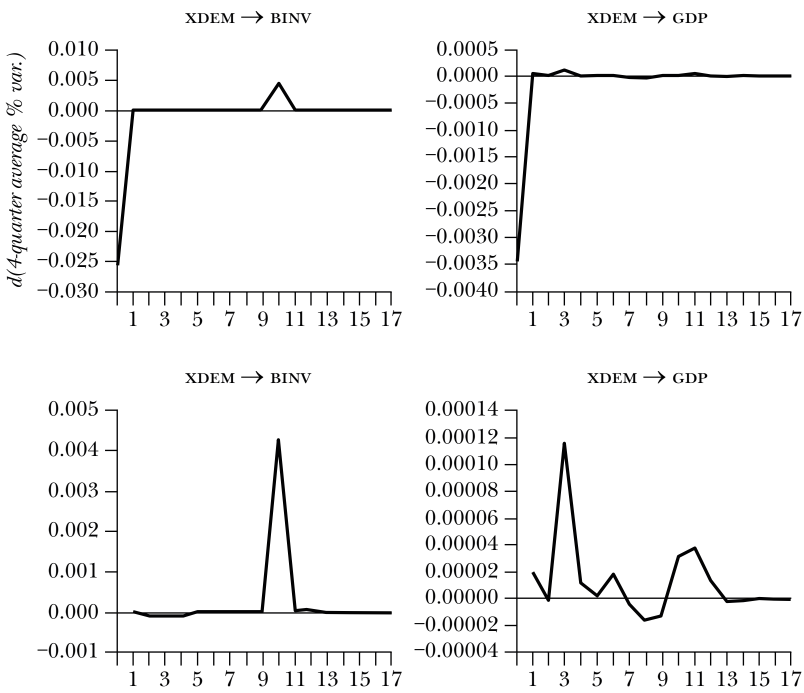


5.5 Impulse Responses in XDem

Figure 7 displays IRFs for the estimated growth in *BInv* and GDP after a shock of 1% in *XDem* growth. In response to a positive structural shock in *XDem* growth, *BInv* growth falls upon impact and then remains unchanged until it increases marginally 10 quarters ahead. Meanwhile, GDP growth decreases almost unnoticeably and subsequently posts a series of modest increases and decreases.

Figure 7

IMPULSE-RESPONSE FUNCTIONS FOR ESTIMATED INCREASES IN BINV AND GDP AFTER A SHOCK OF 1% IN XDEM GROWTH



Given the relative magnitudes, an increase in *XDem* growth is absorbed by a fall in *BInv* growth as well as an imperceptible drop in GDP growth. With respect *BInv* growth, the response is qualitatively similar to the response to a positive structural shock in *PuDem*, meaning it is not possible to reject that the way sales of goods and services are conducted abroad has similar effects to those that stem from the way sales are made to the federal government on aggregate inventory management. In both cases, the magnitude of responses in GDP growth reflects the fact that GDP growth is not the main adjustment channel. Nonetheless, decreases in GDP growth in response to the shock in *XDem* can be understood as the impact of mining production dynamics (where production is reduced when external prices are high).

5.6 Observations

IRF calculations employ a projection without a shock that is not based on a stationary state. A comparison of this projection with the recent execution of aggregate demand components for the 2010Q2-2010Q4 (out of sample) was not encouraging, reflecting that the propagation of shocks during the last two years point to a scenario of an economic slowdown in the medium term.

IRF patterns do not follow a smooth transition as in the over-parameterized linear VAR models. For instance, those for *BInv* are reflected upon impact as well as 10 quarters after the shock to any component of *AgDem* (although with different signs), which is explained by different ways for contracting or demanding goods and services.[29] This lack of a smooth transition is normally obtained when exclusion restrictions are imposed (parsimony) on the parameters of a linear VAR model (see Lütkepohl, 2005). It could also result from the penalized log-likelihood (see Annex B) used when parsimoniously estimating a VARNN-ARCH model.

[29] Another explanation is that mechanisms associated to aggregate inventory management are not reflected so much in their conditional means (that serves to quantify them) as in their conditional second moments. In structural terms, more comprehensive inventory management includes risk factors associated to profits and losses. In econometric terms, it is possible that maximization of the penalized log-likelihood reflects the dominance of changes in the conditional covariance matrix over the quadratic errors of the conditional mean vector.

Finally, the absence of asymmetries in shock response with different signs or magnitudes might be a preliminary but robust result. Optimization of the penalized log-likelihood of a neural network model (see Annex B) is equivalent to a learning process, and this could be lengthy. Due to computing time restrictions, the optimization must be truncated after a large number of iterations, without the network having detected asymmetries. However, the *t*-Student distribution allows for discarding spurious asymmetries in conditional means, making it possible to state that the neural network has still not detected asymmetries in the data because they are not evident.

6. CONCLUSIONS AND OUTLOOKS

This paper econometrically approximates the potentially significant nonlinear effects (asymmetries) that inventory management exerts on production dynamics considering that its volatility varies over time. To that end, we decompose aggregate demand into three components (domestic public, domestic private and external).

The most important results are shown in terms of conditional covariances. Covariances (*BInv*, *PuDem*) and (*BInv*, *XDem*) are negative, reflecting the expected inverse relations when there are no incentives for holding inventories except production smoothing. Covariance (*BInv*, *PrDem*) is the only positive one, reflecting the expected direct relation when there are additional incentives besides smoothing GDP growth. In terms of contemporaneous relations, the only parameter statistically different from zero is consistent with the presence of such additional incentives. This parameter indicates that a positive shock in *PrDem* will be mainly absorbed by *a more than proportional increase* in the production rate shock, meaning there is an amplifier effect (demoderating) of demand shocks on the evolution of production that is explained by the inventory cycle. In fact, some of this faster production rate will be used for increased inventory accumulation, which will probably allow for maximizing profits when current prices are high with respect to the marginal production costs of stocked goods.

Another incentive for holding inventories stems from the need to have a nonfinancial asset that allows for offsetting short-term borrowing incurred to cover production when demand is growing in the event such increased demand reverts. Precisely, given the symmetry found in IRFs, a negative shock in *PrDem* will be offset mainly by a slower rate of production, as well by decreases in the growth of inventory stocks (although to a lesser extent). This result might be consistent with inventory management that takes into account lags in the adjustment of the aggregate production process, as well as changes induced in prices that maximize private profits, particularly when current prices are high compared to the marginal production costs of stocked goods (not necessarily finished goods). In this regard, there are indications that the amplifier effect (demoderating) could be explained by the inventory cycle of raw materials or work-in-progress (although we do not have the data to prove this more specific hypothesis).

The model estimated partly captures episodes around the turning points of GDP in which inventory investment amplifies the response of GDP to large demand shocks. This paper, therefore, provides indirect evidence to support the hypothesis that recent international crises mostly explain recent fluctuations in the inventory cycle (especially for commodity exports) and therefore in the activity of an increasingly globalized economy such as Peru's (see Alessandria et al., 2010). This would provisionally explain the demoderation described in Section 1, particularly in the average incidence of inventory investment growth on real GDP growth during four recently observed recessions in Peru (before that generated as a consequence of the US crisis in 2007-2008; see Table 1).

There is clearly a need to include other potentially relevant variables (some of which are not available for Peru's economy, such as disaggregated inventory investment in raw materials, work-in-progress and finished goods). Given the absence of such disaggregated data, the results of this inventory investment model with aggregate data regarding production *stabilization* could represent a reference for more complete models that manage to include inventories of work-in-progress and raw materials (separate from finished goods) in conditional covariance index modelling. This would provide more appropriate evaluation of production stabilization in terms of conditional second moments, as well as an improvement in the capability of representing the structure of relationships in conditional means and, therefore, in the model's predictive capacity.

ANNEX

Annex A. Aggregate Stock of Inventories according to the Perpetual Inventory Method

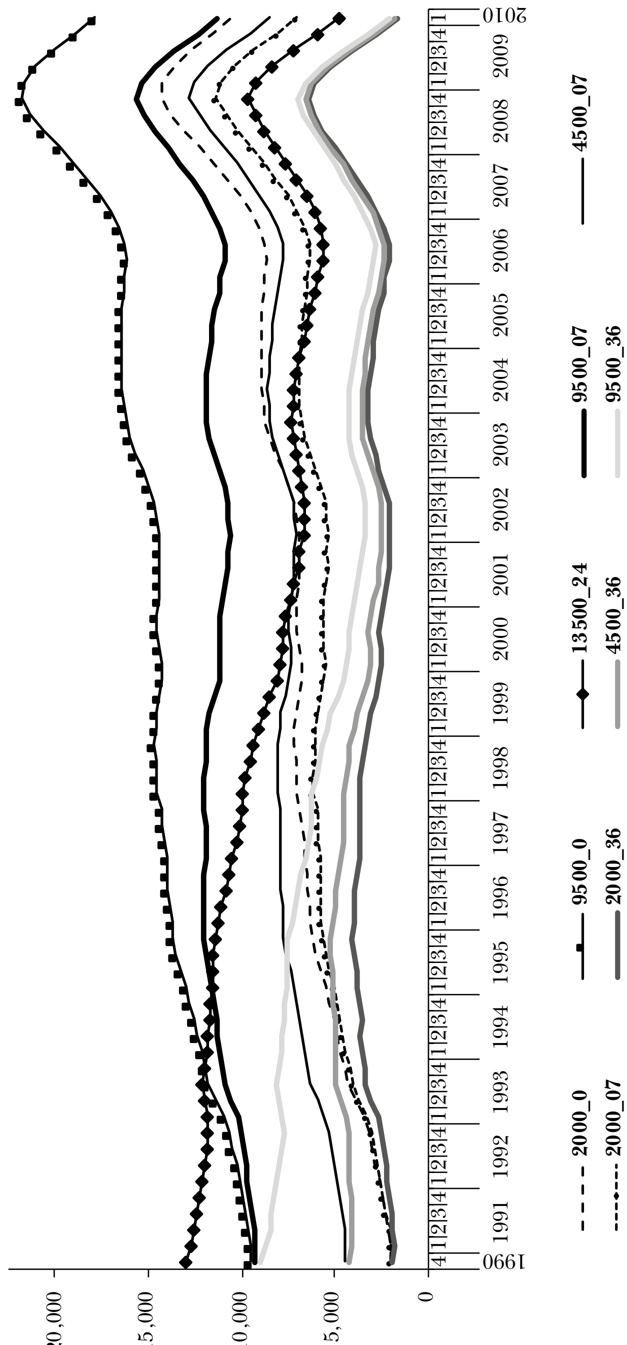
In Peru's experience, shifts in inventory investment have contributed (amplified) recessionary phases since the start of the 1990s. With the US financial crisis (2007-2008), this amplification is more noteworthy, unfolding a demoderation phenomenon in contrast to the Great Moderation observed in the business cycles of the US economy (see introductory discussion). In any case, highly volatile inventory investment growth rates in Peru (see Table 2 in the main text) reveals the need for using a calibrated series of aggregate inventory stocks instead of a series of changes in inventory.

This annex explains the assumptions employed for calibrating a series for the aggregate stock of inventories. This is obtained based on inventory changes data through two quantitative assumptions: *1)* initial inventory stocks and 2*)* the depreciation rate. Figure A.1 presents a set of alternative sequences with initial stocks of between 2,000 million and 13,500 million 1994 soles for the first quarter of 1990, as well as quarterly depreciation rates of between 0.0% and 3.6% (a rate of 2.4% corresponds to that of physical capital that depreciates in 10 years).

Figure A.1

ALTERNATIVE SEQUENCES FOR THE BALANCE OF INVENTORIES IN MILLIONS OF 1994 SOLES

1990Q4-2010Q1



All these inventory stock sequences indicate that, before the international crisis of 2008 affected most economies in the region (2008Q3), Peru had been registering significant inventory accumulation that reached a peak in 2008Q4, just after the initial impact of the crisis was perceived in financial variables such as the exchange rate and interest rates (August 2008). In terms of inventory stocks, the impact of the crisis is evident since the start of 2009 in the form of an unprecedented deaccumulation in the available sample (1990Q4-2010Q1).

All deaccumulations associated to the financial crises of 1995, 1998-1999 and 2001 appear small in size and generally affect the evolution of inventory stocks cumulatively, for instance, when assuming a depreciation rate higher than that for physical capital (for instance, with a quarterly rate of 3.6%) and 2,000 million or 4,500 million of initial stock. If we wish to reduce the preponderance of the sharp accumulation and later deaccumulation of inventories associated to the international crisis of 2008 in the sample, the initial stock can be raised slightly to above 5,000 million, which would be qualitatively compatible with high inventory levels expected to be registered at the start of the 1990s.[30] This paper explicitly addresses the conditionality of all the results with respect to these two quantitative assumptions: *1)* initial inventory stock and *2)* depreciation rate.[31]

[30] Fujino (1960) refers to high levels of inventory stocks of finished goods as a percentage of demand in some Japanese industries in 1950 or 1951 due to speculation under the setting of the Korean war (June 1950 to July 1951). Japan provided military, logistical and medical support to the allied forces led by USA.

[31] The results shown use a calibrated sequence of inventory stocks that assumes an initial balance of 2,000 million 1994 nuevos soles and a null depreciation rate (perpetual inventories).

Annex B. Estimation via Penalized Maximum Likelihood

Estimation of multiple time series models typically finds the problem of over parameterization unsurmountable. The usual strategies for tackling this problem have been elimination algorithms with stepwise and a data criteria sequence, thereby achieving parsimonious models.

Based on statistical applications to penalized regression problems in chemistry and biology (molecule and genotype structures), the literature on parameter shrinkage has re(emerged); in it a penalization function in them, is included which is added to the function that typically optimized in parameter estimation (GLS, GMM or MV).[32]

In the case of MV estimation, the loss function minimized is the negative of log-likelihood, which we denote as $L(\theta)$, where θ is a parameter vector. In a system with multiple variables, this vector θ can be decomposed into two blocks: interceptors α and all other parameters β, to define the penalized loss function as

B.1 $$g(\theta) \equiv L(\theta) + P_{\lambda}(\beta),$$

where $P_{\lambda}(\beta)$ is one of the three penalized functions available in the literature (see McCann and Welsch, 2006, and Ulbricht and Tutz, 2007) that depend on tuning parameters λ_i (positive):

[32] The typical MCO estimator minimizes $SSE(\tilde{\beta}) \equiv (y - x\tilde{\beta})'(y - x\tilde{\beta})$. To avoid a potential problem of multicollinearity, the ridge estimator $\tilde{\beta} \equiv [x'x + \lambda Q]^{-1} x'y$ was devised to minimize $SSER(\tilde{\beta}) \equiv SSE(\tilde{\beta}) + \lambda\tilde{\beta}'Q\tilde{\beta}$, where Q should be a positively defined arbitrary matrix and $\lambda > 0$ so the MCO estimator *regularizes* (see Firinguetti and Rubio, 2000, for references and a generalization). Returning to our context, a parsimonious estimator belongs to this same family of estimators because $Q = I$ obtains the penalized version of $SSE(\tilde{\beta})$.

1) Lasso or *L1* (strong zeros; Tibshirani, 1996), $P_\lambda(\beta) \equiv \lambda \sum_{i=1}^{q} |\beta_i|$.

2) Ridge or *L2* (against over-parameterization), $P_\lambda(\beta) \equiv \lambda \sum_{i=1}^{q} \beta_i^2$.

3) Elastic network (*L1* and *L2*), $P_\lambda(\beta) \equiv \lambda_1 \sum_{i=1}^{q} |\beta_i| + \lambda_2 \sum_{i=1}^{q} \beta_i^2$.

The most direct reason for optimizing this new loss function is clearly that of estimating the parameters at the same time as selecting the specification (Fan and Li, 1999). This model selection is apparently more direct than the alternative of performing a series of hypothesis tests. Nonetheless, the main motivation is to reduce the mean squared error (MSE) of the sample. One well-known econometric result is that the MV estimator over-estimates the length of the true parameter vector when the regressors are not orthogonal amongst each other, causing significant bias in the MV estimator. Minimizing this bias led to the family of *ridge* estimators (see Fomby et al., 1984, pp. 300-302 and references), specifically an MV estimator with restrictions or penalties.

However, similarly to the *ridge* family of estimators (see note 29), it is necessary to determine tuning parameters $\lambda > 0$ through a set of estimations for different values of λ.[33]

B.1 Tuning λ Parameters in VARNN-ARCH Models

We define the estimator we will use as

B.2
$$\theta(\lambda) \equiv \arg\min\{g(\theta)\}.$$

[33] The complexity of the resulting optimization problem for each fixed value of λ is considerably greater, meaning addressing it various times to fill a grid and thereby select the tuning parameters (and associated β parameters) is extremely costly in computational terms. For the simple case of a *lasso* regression, a group of algorithms has been proposed (see Wu and Lange, 2008).

Tuning parameters λ are basically Lagrange multipliers and are usually determined in such way that the asymptotic mean squared error (MSE) of estimator $\theta(\lambda \neq 0)$ is less than the asymptotic variance of the estimator of MV, $\theta(\lambda = 0)$. This determination is direct in a simple problem such as a linear regression, but generally requires, in the case of the *elastic network*, a search algorithm in an $\mathbb{R}^2_{++}$ mesh with simulation at each point of it, a procedure too computationally costly for a VARNN-ARCH model.

The alternative is to define its optimization as a *weak apprentice*, i.e., (λ_1, λ_2) with large values to force small changes in each maximum likelihood iteration and thereby obtain more stable estimates (Ulbricht and Tutz, 2007).[34] The advantage of this likelihood penalization is that neural network training and pruning is performed in parallel, meaning the neural network can adapt for minimizing errors associated with pruning (see Reed, 1993). This alternative was the first to be used for a VARNN-ARCH model, without managing to converge after a large number of iterations.

After forcing very small changes with large values for (λ_1, λ_2), we used ad hoc values based on the proposals of Fan and Li (1999), i.e.,

B.3 $$\lambda_i = \sqrt{2\log(nparam)},$$

where *nparam* is the total number of θ parameters in the model. This strategy did not manage convergence for an even higher number of iterations (three million). The results reported in this version of the paper use nonstructural parameters of the VARNN-ARCH estimated using this strategy.

[34] In fact, the nonlinear classification problems that are typical applications of neural networks, optimization of the objective function $L(\theta)$ is established around a set of desirable values, defining these regularization penalties and fixing parameters (λ_1, λ_2) through other criteria. See Jaakkola (2006).

B.2 Alternative to a Single Tuning Parameter

Finally, results were obtained with the truncated maximum (reaching the maximum number of iterations without converging) of the penalized likelihood function for a lasso function using the value of the previous equation for the single tuning parameter. These results have allowed for estimating the proposed contemporaneous structure and performing *provisional* tests on it (they would not be provisional if the required convergence had been achieved), which has been reflected in a lack of accuracy of the projections generated. Although convergence has not been produced after a prohibitive number of iterations, in this subsection we present an alternative tuning strategy proposed by Wang et al. (2007).

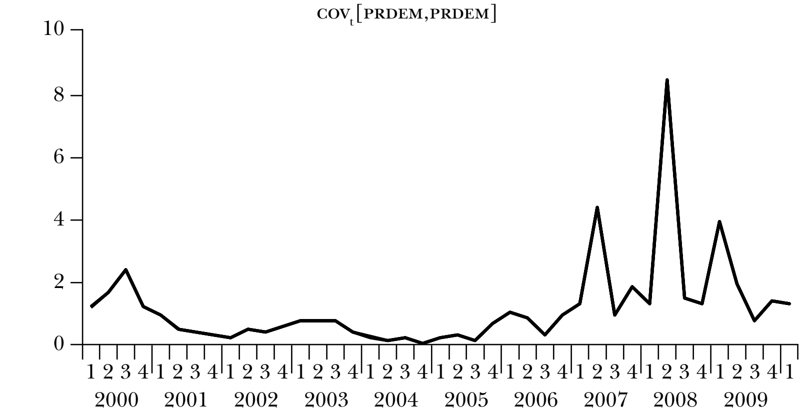
Wang et al. (2007) propose discarding the lasso penalty with a single tuning parameter due to the potentially significant bias it generates and using multiple tuning parameters, in fact, one for each parameter of the unpenalized likelihood function.

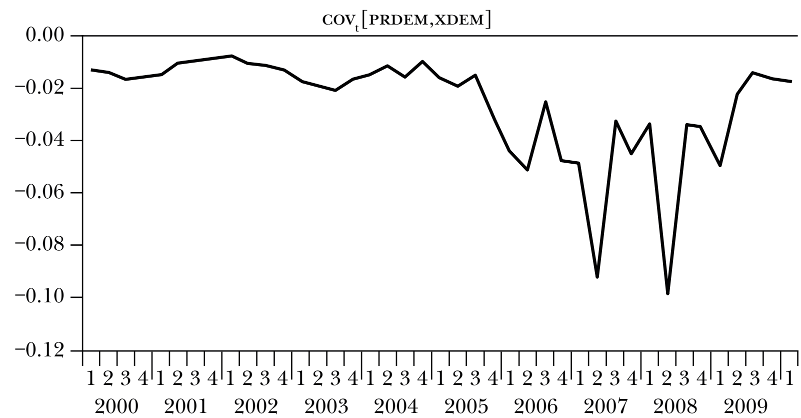
B.4
$$\lambda_i = \frac{\log(nparam)}{nparam\lambda_i}.$$

Greater parametric complexity in the penalization function proposed in Wang et al. (2007) is addressed through a profitable strategy for estimating in a first stage all the tuning parameters for optimizing the unpenalized likelihood, and then using said estimates in a second stage of penalized likelihood optimization. Another advantage of this strategy is that it solves the problem of a lack of asymptotic characteristics required for performing statistical tests when there is only one tuning parameter.

Annex C. Evolution of Conditional Covariances (Standardized)

Figure C.1





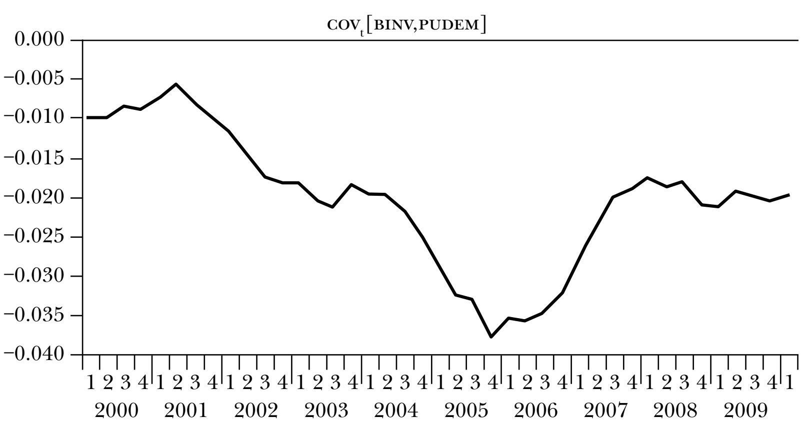


Figure C.1 (cont.)

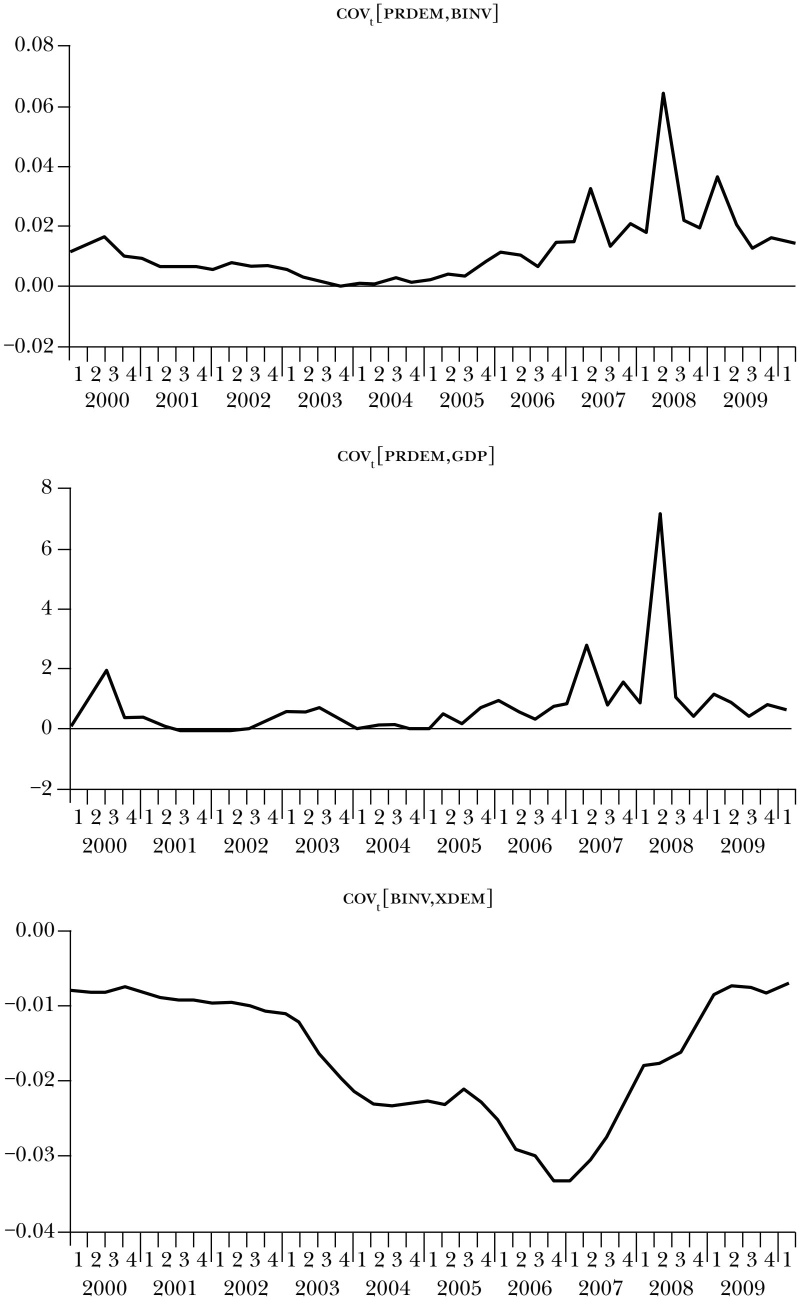


Figure C.1 (cont.)

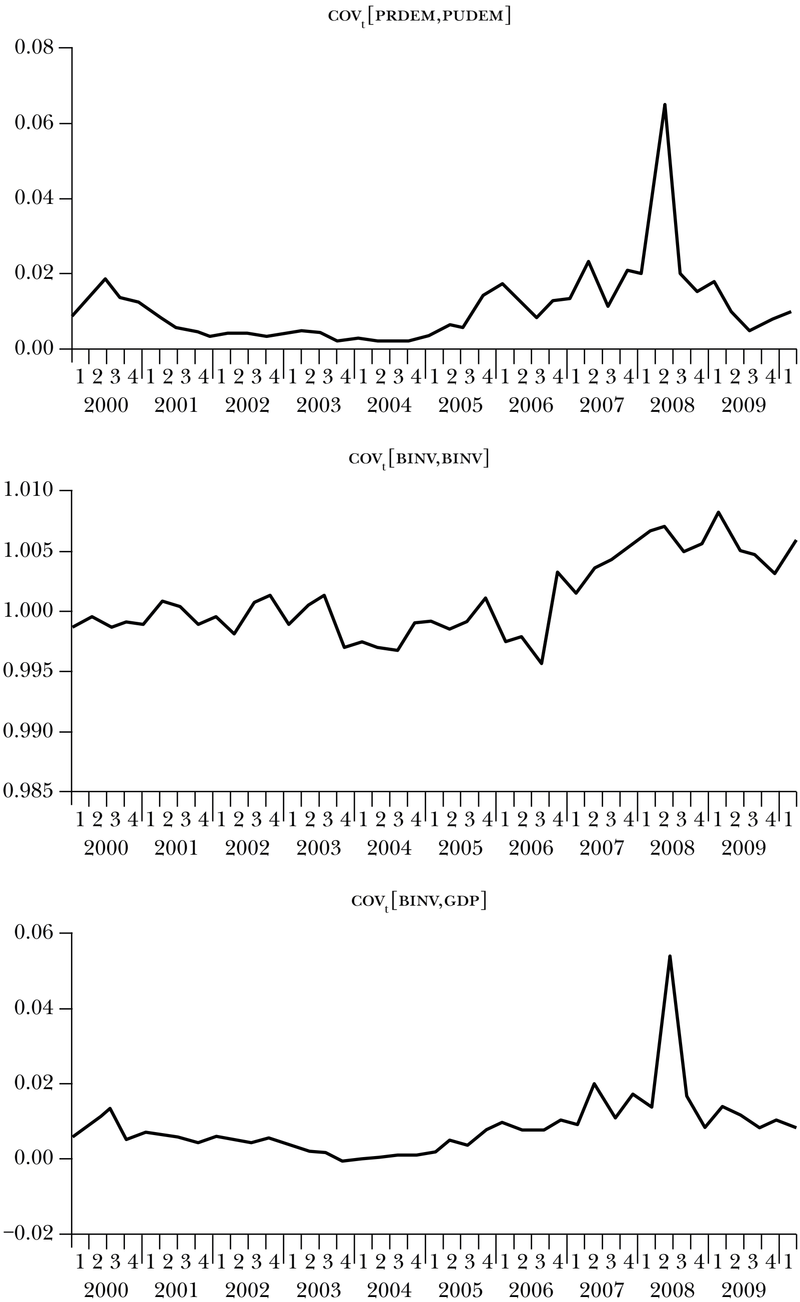


Figure C.2

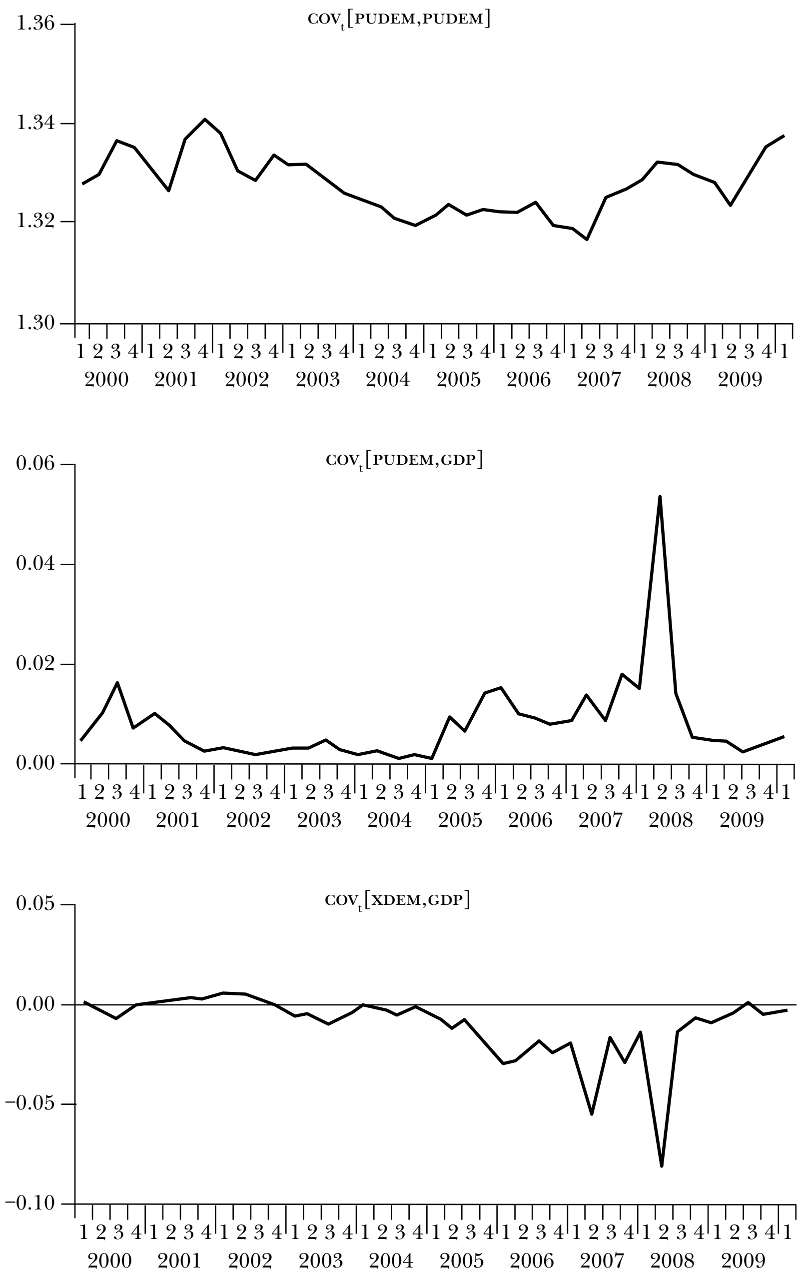
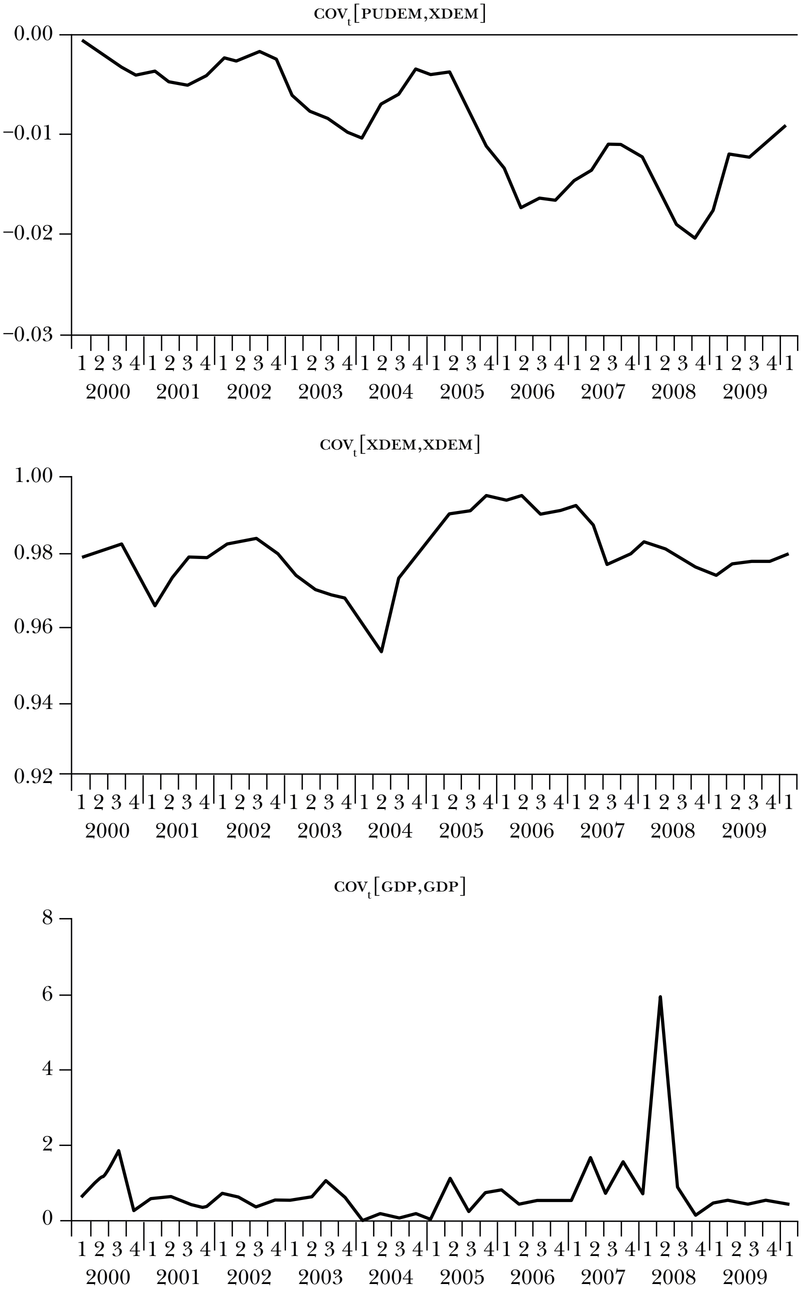

cov$_t$[pudem,pudem]
1.36
1.34
1.32
1.30
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009
cov$_t$[pudem,gdp]
0.06
0.04
0.02
0.00
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009
cov$_t$[xdem,gdp]
0.05
0.00
−0.05
−0.10
1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1
2000 2001 2002 2003 2004 2005 2006 2007 2008 2009

Figure C.2 (cont.)



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