Roberto Bonilla
Alberto Trejos

Marriage, Employment Participation and Home Production in Search Equilibrium

Abstract

We model a marriage market where singles consider the prospects of employment and income of their potential spouses, and married couples make joint decisions on home production and labor participation. This double interaction between the marriage and labor markets is affected by search frictions in both. We characterize the job search strategies of different couples; equal individuals have different behaviors depending on their spouses. When the search for mates is easy, people marry others with very similar productivity, and both spouses have the same behavior in the labor market. This natural outcome is socially inefficient as it takes some high productivity people off the labor market and viceversa. It also expands income distribution. Some empirical findings in the labor literature are supported theoretically here.

Keywordds: labor markets, marriage markets, employment, home production, equilibrum.

JEL classification: D83, J12, J21.

Roberto Bonilla, professor, Business School, Newcastle University, United Kingdom. Alberto Trejos, professor, Business School, INCAE.

1. INTRODUCTION

In any labor market, there is heterogeneity in workers' wages and labor participation rates. Part of it is explained, of course, by differences in productivity: more productive individuals are likelier to pursue work, and to receive a better wage. But, in general, other circumstances may also matter. In particular, the employment status, prospects and wages of one's spouse may affect whether one seeks work, as one shares income and efforts toward home production with that spouse. Married to a high earner, one is likelier to engage in home production, or to be more selective about which jobs to accept. Interestingly, since a person can get clear indications about a marital partner's earning potential while pondering the possibility of marriage, then not only their career is affected by the productive features of their spouse, but their choice of spouse is also affected by their potential careers.

In this paper, we develop a model where agents go first to a marital market and then a labor market. Agents choose their spouses taking into account their expected earnings, and once married the couple makes joint decisions about job search. Hence, the two-directional interaction described above is brought forward. Furthermore, spouses can collaborate not only by working and sharing their income, but by specializing, one in market work, and the other in home production.

We find that in equilibrium, across the space of all possible couples, each pair of spouses has a unique optimal strategy of labor search. There is a positive correlation in earning potentials among spouses. When frictions in the marriage market are small, this correlation is very tight. Couples where both spouses have very similar productivity also have symmetric (within the couple) labor search strategies. Very heterogeneous couples behave asymmetrically. In equilibrium, the population is divided in four classes: spouses with a similar (and high) productivity will constitute a high class where both will always stay in the job market, eventually sacrificing home production completely. If their productivity is similar but lower, they will choose to take turns to work, and at most generate one income. Other more heterogeneous couples will display strategies where the more productive member is always in the market, and the less productive one stays at home always, or almost always.

Theoretically, our paper contributes to the growing literature that studies the interaction between the marriage and labor markets. We expand on Violante et al. (2012), who show how reservation wages are affected by marital status and joint search. Jaquemet and Robin (2013) study individual labor supply with a frictional marriage market. Bonilla and Kiraly (2013) study how the marriage wage premium arises as an equilibrium outcome in a model with frictional labor and marriage markets. Bonilla et al. (2015) study the link between marriage and beauty wage premia in search equilibrium. We add to this literature as our main purpose is to study the consequences of the link between search for a partner and the facts that participation in the labor market is optional and consumption is, at least partially, a public good.

Empirically, the main contribution of the paper is to explain many previously documented facts from a coherent theoretical framework. Our results reflect Schwartz (2010) who convincingly document that as the search technology has improved, the positive correlation in earning potential among spouses has increased, raising overall income inequality. This increased symmetry in the human capital that the spouses bring into the household reflects an increasing similarity in their inputs and home production hours, as has been shown as far back as Cancian et al. (1993). Schwartz and Mare (2005) analyzes the data and reaches conclusions about the assortative nature of spouse choices, and about the implicit participation decisions, that interestingly fit our main theoretical results. We also get a theoretical explanation for Powell (1997) and Lovász and Szabó-Morvai (2014), who find a positive effect of improved child care provision on female labor supply.

We describe the environment in Section 2, and derive the equilibrium in Section 3. We conclude in Section 4.

2. THE ENVIRONMENT

Time is continuous and continues forever. The population is a continuum of measure Ω_w of infinitely lived women, and another of measure Ω_m of infinitely lived men. Men and women discount future consumption at rate *r*. Each agent is characterized by an observable productivity $p \in \left[\underline{p}, \overline{p}\right]$, taken from the distribution function $F_m\left(p\right)$ in the case of men and $F_m\left(\overline{p}\right)$ in the case of women.

When young, agents first enter a marriage market, where they can search (at a minimal but positive search cost) and encounter members of the opposite sex. For two people to be able to marry they require to be compatible (that is, all aspects of the relationship besides work and income, like attraction, personality, etc.), and not all potential couples are such. We assume that compatibility is a binary characteristic of the couple rather than the individual, uncorrelated with productivity, and not a matter of degree (in other words, if I like you then you like me, and while we could both also like others out there, we would not like them more or less). These meetings between men and women emerge through a Poisson process. For a searching man, compatible women are encountered with an arrival rate $\mu_m = \mu\Omega_w$, and women find compatible men with arrival rate $\mu_w = \mu\Omega_m$.

Upon meeting a potential compatible candidate of the opposite sex, agents also observe their productivity, and they then decide whether to enter a permanent monogamous relationship, which emerges if doing so is strictly mutually agreeable; otherwise, they keep searching for another spouse.[1] We also

[1] The case of unobservable productivity could be interesting, but is not central to the main point of the paper. In addition, this has been addressed in the literature on frictional labor and marriage markets (see for example Boulier and Rosenzweig, 1984; Masters, 2008, or Brien et al., 2006). The most obvious consequence of introducing such consideration is that, in some of the couples, the partners realize ex post that they are not mutually desirable. This would shift the attention to the possibility of divorce, which is not the main concern of the paper. This would require to model in much more detail the process by which information is revealed.

assume that agents can only entertain one suitor at a time, and need to give up a match in order to encounter other matches. When a couple marries, two clones of the newlyweds take their place in the marriage market.

Only once they have married do agents enter the labor market. When searching, jobs are found at another Poisson process, with arrival rate λ. Jobs pay as wages the worker's marginal productivity, and are *indivisible* – or exclusively full-time – employment, in the sense that the number of hours worked is not variable.[2] With arrival rate $\delta > 0$, the job exogenously ends.

Spouses share income and home production; once married, preferences correspond to the couple, not the individual spouses. The value of home production includes two components. The first one (denoted h) is independent of couple's income and is enjoyed when at least one of the partner's is unemployed. The second component increases with income with a marginal effect denoted α (that, for instance, enables to acquire household goods that complement home work or enhance its enjoyment), but requires that at least one member of the couple is not working (to produce that home work). A second unemployed member of the family would be a waste, generating no income and adding nothing to the value of home production. Hence, instantaneous tow utility is:

$$U = \begin{cases} p + P & \text{both work and earn } p \text{ and } P \\ p + h + \alpha p & \text{if one works for wage } p \text{ and the other} \\ h & \text{does not work neither is employed.} \end{cases}$$

In the appendix we address the link between this indirect utility function and the direct function more commonly used in the literature.

---

[2] Here, as in much of the macro literature and also, for instance, in Rogerson and Wallenius (2012), individuals either work full time or not at all. Making the number of hours worked an endogenous variable (as in Rogerson and Wallenius, 2013) will probably not change the general meaning of the main results.

Searching for production opportunities carries a cost ε; we assume $\varepsilon > 0$ but look at the limit case where $\varepsilon \to 0$. This infinitesimal cost implies that agents will search only when they expect a strictly positive surplus from the market.[3]

3. EQUILIBRIUM

Due to the sequential nature of the problem, we can work out the labor market choices and performance of any possible couple (whether in equilibrium such a couple would exist or not). Then, given the benefits obtainable in different matches, we look at the spouse-searching strategies of men and women.

For now, with no loss of generality, we will label H the spouse with a weakly higher productivity and L the other spouse; whether H is the man or the woman will of course vary across couples, and is irrelevant for now. Their productivities will be denoted p_H and $p_L \leq p_H$. The value functions that correspond to their circumstances are denoted V_{HL}, where H (or L) take the value 1 when the spouse H (or L) has a job, and 0 when not. These functions V_{HL} are specific to each couple, as another pair with different productivities would enjoy different returns. Then,

1
$$\begin{aligned}
rV_{00} &= h + \phi_0\lambda\left(V_{10} - V_{00}\right) + \phi_1\lambda\left(V_{01} - V_{00}\right) \\
rV_{10} &= \left(1+\alpha\right)p_H + h + \delta\left(V_{00} - V_{10}\right) + \phi_2\lambda\left(V_{11} - V_{10}\right) \\
rV_{01} &= \left(1+\alpha\right)p_L + h + \delta\left(V_{00} - V_{01}\right) + \phi_3\lambda\left(V_{11} - V_{01}\right) \\
rV_{11} &= p_H + p_L + \delta\left(V_{01} + V_{10} - 2V_{11}\right).
\end{aligned}$$

---

[3] Assuming costly search will keep us from having to analize mixed-strategy equilibria where agents who are indifferent between going to the market or not randomize the decision. Here, if you are indifferent about the outcome of search, you choose not to search, to avoid the cost. Notice we do not need to make the same assumption of costly search for the marriage market. In fact, the proofs below are cleaner when we assume that men or women who are strictly indifferent between accepting or rejecting a particular partner always reject and keep searching, for an alternative the leaves them strictly better off.

The first equation tells us that the flow value of a couple where neither has a job is given by the value of home production (which, when nobody is getting an income, is just h), plus two factors related to their search behavior. First, if H is searching (with probability ϕ_0) the arrival rate λ of production opportunities that deliver the surplus $V_{10} - V_{00}$. Second, if L is searching (with probability ϕ_1), the arrival rate λ times the surplus $V_{01} - V_{00}$. The second equation tells us that a couple where only H works enjoys income p_H, plus the fruits of the home production of L (augmented by the income generated by H, or $h + \alpha p_H$), plus the arrival δ of the destruction of H's job, times the implied net loss $(V_{00} - V_{10})$, plus, if L is searching for a job (with probability ϕ_2), the arrival λ of the surplus $V_{11} - V_{10}$. The other two equations can be understood analogously, given ϕ_3 is the probability H would search for a job when L is working.

For couples to behave optimally, at every chance they only search for a job if it improves their condition, so:

$$\phi_0 = \begin{cases} 1 & \text{if} \quad V_{10} > V_{00} \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_1 = \begin{cases} 1 & \text{if} \quad V_{01} > V_{00} \\ 0 & \text{otherwise} \end{cases}$$

2

$$\phi_2 = \begin{cases} 1 & \text{if} \quad V_{11} > V_{10} \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_3 = \begin{cases} 1 & \text{if} \quad V_{11} > V_{01} \\ 0 & \text{otherwise} \end{cases}$$

Definition 1: For all couples (H, L) an optimal job-search strategy is a combination of values $V = (V_{00}, V_{01}, V_{10}, V_{11})$ and labor search probabilities $\phi = (\phi_0, \phi_1, \phi_2, \phi_3)$ that satisfies the Bellman equations (1) and incentive compatibility conditions (2).

In principle, ϕ could take 16 different values, but the set of possible situations narrows quite a bit thanks to the following:

Lemma 2: In any optimal strategy, *a)* $\phi_0 = 1$, and *b)* $\phi_2 = 1 \Rightarrow \phi_1 = \phi_3 = 1$.

Proof. Recall that there are no mixed strategies so the values $\phi_i \in \{0,1\}$. *a)* Clearly, $\phi_0 = 0$ cannot be an optimal strategy when $\phi_1 = 0$, because the couple can raise their income with no sacrifice in home production if at least one of the members gets a job. On the other hand, $\phi_0 = 0$ while $\phi_1 = 1$ cannot be the optimal strategy since it is dominated by $\phi_0 = 1,\ \phi_1 = 0$. Hence, there are no optimal strategies where $\phi_0 = 0$. *b)* When $\phi_2 = 1$, the lower-productivity spouse searches even though the higher-productivity spouse is employed. Then, the option of gaining p_L is worth giving up $h + \alpha p_H$. Clearly, if this is the case, it is also optimal for H to search when only L is working since $p_H > p_L$ (the prize is higher) and $h + \alpha p_L < h + \alpha p_H$ (the sacrifice is lower). Hence, $\phi_2 = 1 \Rightarrow \phi_3 = 1$.

If the option of gaining p_L (the same prize) is worth giving up $h + \alpha p_H$ (because $\phi_2 = 1$), then it is also worth giving up just h. Hence, $\phi_2 = 1 \Rightarrow \phi_1 = 1$. ■

The lemma indicates that, of the 16 possible combinations of ϕ_i that constitute alternative values for $\phi = (\phi_0, \phi_1, \phi_2, \phi_3)$, the eight that include $\phi_0 = 0$ are not the best strategy for any couple, nor the three that include $\phi_2 = 1$ and either $\phi_1 = 0$ or $\phi_3 = 0$. Of the remaining five options, two [$\phi = (1,0,0,0)$ and $\phi = (1,0,0,1)$] can be collapsed into one –say $\phi = (1,0,0,\cdot)$ since they only differ in ϕ_3 which describes a choice that only happens if the L has a job, something that does not emerge on the equilibrium path if $\phi_1 = 0$ and $\phi_2 = 0$. Therefore, there are at most four possible types of optimal strategies, where $\phi = (\phi_0, \phi_1, \phi_2, \phi_3)$ assumes the values (1,0,0, ·), (1,1,0,0), (1,1,0,1) or (1,1,1,1).

The procedure to find out when each of these search strategies is the couple's optimal behavior is very simple: Assume one of the four candidate values of ϕ and substitute it in 1; then, solve for V_{HL}, and finally verify the parameter combinations for which 2 hold for that candidate ϕ, given those solutions for V_{HL}.

Let's analyze first the strategy (1,1,1,1) where both spouses are always on the job market –either working or searching for work. This strategy leads to families that (in their preferred state) generate two incomes but no home production. Substituting $\phi = (1,1,1,1)$ in 1 and solving for V_{HL} yields:

$$\begin{aligned}\Gamma V_{00} &= \lambda\left[r(1+\alpha)+2(\delta+\alpha\delta+\lambda)\right](p_H+p_L)\\ &\quad +2\delta\left(r^2+3r(\lambda+\delta)+2\delta(\delta+2\lambda)\right)h\\ \Gamma V_{10} &= r\left[(1+\alpha)r+\left(2(\alpha+1)\delta+(2\alpha+3)\lambda\right)\right]p_H\\ &\quad +\lambda\left(2(\alpha\delta+\delta+\lambda)+r\right)(p_H+p_L)\\ &\quad +(2\delta+r)(\delta+2\lambda+r)h\\ \Gamma V_{01} &= \left[(\alpha+1)r^2+r\left(2(\alpha+1)\delta+(2\alpha+3)\lambda\right)\right]p_H\\ &\quad +\lambda\left(2(\alpha\delta+\delta+\lambda)+r\right)(p_H+p_L)\\ &\quad +(2\delta+r)(\delta+2\lambda+r)\,h\\ \Gamma V_{11} &= \left[r^2+r\left((2+\alpha)\delta+3\lambda\right)+2\lambda(\delta+\alpha\delta+\lambda)\right](p_H+p_L)\\ &\quad +2\delta(r+\delta+2\lambda)h\end{aligned}$$

where $\Gamma = r(\delta+\lambda+r)\left(2(\delta+\lambda)+r\right)$.

Although these expressions are messy, they are also straightforward, and one can apply them to derive that

$$V_{01} > V_{00} \leftrightarrow p_L > \frac{\lambda(h+\alpha p_H)}{2(\alpha+1)\delta+(\alpha+2)\lambda+(\alpha+1)r}.$$

Similarly

$$V_{11} > V_{10} \leftrightarrow p_L > \frac{(\delta+2\lambda+r)(h+\alpha p_H)}{(\alpha+2)\delta+2\lambda+r} \equiv g_1(p_H)$$

and

$$V_{11} > V_{01} \leftrightarrow p_L < \frac{\left((\alpha+2)\delta+2\lambda+r\right)p_H-(\delta+2\lambda+r)h}{\alpha(\delta+2\lambda+r)}.$$

A little further exploration confirms that the first constraint is not binding, because it is laxer than the second constraint for all p_H. Furthermore, the second and third conditions coincide on the (p_H, p_L) plane on the 45° line, at the value

$$p_H = p_L = p^* = \frac{(\delta+2\lambda+r)h}{(1-\alpha)r+2\left((1-\alpha)\lambda+\delta\right)}.$$

For $p_H < p^*$, the two conditions cannot be satisfied jointly. For $p_H \geq p^*$, the third constraint is redundant with $p_L \leq p_H$. Hence, the strategy $\phi = (1,1,1,1)$ is only the optimal job-strategies for couples where $p_H > p^*$ and $p_L > g_1(p_H)$. In other words, this is the behavior in couples where both spouses have similar (and high) productivity.

Analogous derivations can be obtained to characterize for which couples is each of our remaining options of ϕ the optimal job-search strategy. This analysis is presented in the Appendix, and its results are summarized in the following:

Proposition 3. For all possible couples $(p_H, p_L) \in \left[\underline{p}, \overline{p}\right] \times \left[\underline{p}, p_H\right], \exists! \phi = (\phi_0, \phi_1, \phi_2, \phi_3)$ that is optimal, In particular, there are values $p^*, p^o < p^*$ and (linear, increasing) functions $g_i(p_H)$, $i \in \{1, \ldots, 4\}$ such that the optimal job-search strategy is

a) $\phi = (1,1,1,1)$ if $p_H > p^*$ and $p_L > g_1(p_H)$.

b) $\phi = (1,1,0,0)$ if $p^o < p_H \leq p^*$ and $p_L > g_3(p_H)$ or if $p_H \leq p^o$ and $p_L > g_2(p_H)$.

c) $\phi = (1,1,0,1)$ if $p_H > p^*$ and $p_L \in \left(g_4(p_H), g_1(p_H)\right]$, or if $p_H \in \left(p^o, p^*\right]$ and $p_L \in \left(g_4(p_H), g_3(p_H)\right]$.

d) $\phi = (1,0,0,\cdot)$ if $p_H \leq p^o$ and $p_L \leq g_2(p_H)$ or if $p_H > p^o$ and $p_L \leq g_4(p_H)$.

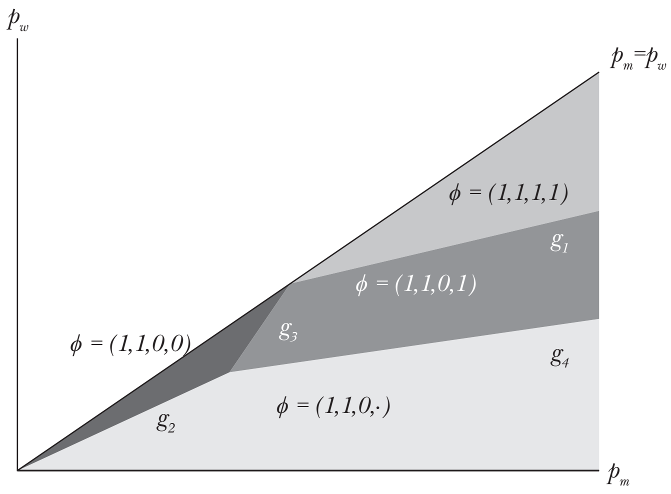
Figure 1 illustrates these results.

Lemma 4. A couples utility is an increasing function of the productivity of each of its members. That is, $V_{0,0}\left(p_H = \max\{p, \pi\}, p_L = \min\{p, \pi\}\right)$ is a weakly increasing, piecewise linear, weakly convex function of p, given π with slope 0 at $p = 0$.

Proof. Let $\pi < p^o$. For $p \sim \underline{p}$, $\partial V_{0,0}\left(p_H = \max\{p, \pi\}, p_L = \min\{p, \pi\}\right) / \partial p = \partial V_{00}^{1000} / \partial p_L = 0$. For $p > g_2(\pi)$ that derivative becomes $\partial V_{00}^{1100} / \partial p_L > 0$, then for $p > \pi$ it is $\partial V_{00}^{1100} / \partial p_H = \partial V_{00}^{1100} / \partial p_L$, then for $p > g_2^{-1}(\pi)$ it is $\partial V_{00}^{1000} / \partial p_H > \partial V_{00}^{1100} / \partial p_H$. All these derivatives are non-negative and constant, and each is larger than the previous one.

Figure 1

OPTIMAL STRATEGIES FOR COUPLES



To verify the same for $\pi \in \left[p^o, p^*\right]$, simply it is a matter of verifying that $\partial V_{00}^{1101} / \partial p_H > \partial V_{00}^{1100} / \partial p_H = \partial V_{00}^{1100} / \partial p_L > \partial V_{00}^{1101} / \partial p_L > \partial V_{00}^{1000} / \partial p_L$, the appropriate values for $\partial V_{0,0}\left(p_H = \max\{p, \pi\}, p_L = \min\{p, \pi\}\right) / \partial p$ as p moves from $\left[\underline{p}, g_4(\pi)\right]$ to $\left[g_4(\pi), g_3(\pi)\right]$ to $\left[g_3(\pi), \pi\right]$ to $\left[\pi, g_3^{-1}(\pi)\right]$ to $\left[g_3^{-1}(\pi), \overline{p}\right]$. Finally, to check the case where $\pi > p^*$, verify $\partial V_{00}^{1101} / \partial p_H > \partial V_{00}^{1111} / \partial p_H = \partial V_{00}^{1111} / \partial p_L > \partial V_{00}^{1101} / \partial p_L > \partial V_{00}^{1000} / \partial p_L$, which in turn corresponds to $\partial V_{0,0}\left(p_H = \max\{p, \pi\}, p_L = \min\{p, \pi\}\right) / \partial p$ as p moves from $\left[\underline{p}, g_4(\pi)\right]$ to $\left[g_4(\pi), g_1(\pi)\right]$ to $\left[g_1(\pi), \pi\right]$ to $\left[\pi, g_1^{-1}(\pi)\right]$ to $\left[g_1^{-1}(\pi), \overline{p}\right]$. ■

Notice we find that symmetric couples have symmetric strategies, and viceversa, in the sense that when the difference in productivity between husband and wife is small, the optimal job search behavior is the same for both.

Notice, for instance, the two regions adjacent to the 45° line, where $p_L \sim p_H$. At the top, a marriage of two similarly productivity people keeps them in the labor market all the time. At the bottom, a marriage of similarly (low) productivity people keeps one of them – does not matter which – at home; when both are unemployed, both search, and when either one finds a job, the other one stops searching. Meanwhile, in the other regions, and especially in the region below where p_L is very low, the differences between the spouses are large, and their behavior is asymmetric. For instance, in the $\phi = (1,0,0,\cdot)$ region, one spouse is always in the market and the other is always at home.

Our results reflect the pattern identified in Powell (1997) and Lovász and Szabó-Morvai (2014) who find that more accessible child care provision, by lowering the opportunity cost of home production, increases female labor supply. Think for example of decrease in the value of home production. In this case $g_3(p_H)$ shifts to the left, while $g_4(p_H)$, $g_1(p_H)$ shift down and both p_0 and p^* decrease. Further, the decrease in $g_1(p_H)$ is higher than that of $g_4(p_H)$. This translates into the following qualitative results: The area $\phi = (1,1,0,0)$ and $\phi = (1,0,0,\cdot)$ both decrease, reflecting that now fewer marriages will be such that one of the partners (the L in area $\phi = (1,0,0,\cdot)$) ends up not participating in the labor market. The area $\phi = (1,1,0,1)$ also decreases, but it does so to accommodate the expansion of area $\phi = (1,1,1,1)$. Once again, this means less partnership in which the L stops participating in the labor market, and more marriages in which both partners remain in the labor market forever. A similar pattern obtains as one analyzes a decrease in α (brought about for example by an increase in the social provision of leisure opportunities).

As frictions disappear in the labor market $(\lambda \to 0)$, we find that $p^o = p^* = \dfrac{h}{1-\alpha}$, $g_4(p_H) = g_1(p_H) = h + \alpha p_H$, and $g_2(p_H) = p_H$. This means that areas $\phi = (1,1,0,0)$ and $\phi = (1,1,0,1)$ disappear. In other words, the only couples that may arise are either those where both spouses work or those were L never works; especially the latter are very frequent for extremely high values of λ, since $\frac{\partial p^*}{\partial \lambda} > 0,\ \frac{\partial g_1(p_H)}{\partial \lambda} > 0$.

The results also match the findings in Schwartz (2010) that improvements in the partners' search technology increases the association between spouses' earnings, thus raising inequality as marriages increasingly consist of two high-earning or two low-earning partners.[4] In our model, the individual behavior of different types of couples may augment this inequality across society. See, for example, the contrast between a couple applying $\phi = (1,1,1,1)$ and another choosing $\phi = (1,1,0,0)$. Individually, each member in the first couple is more productive than each member in the second. Collectively, when both couples reach their desired state the former has twice the number of employed people than the latter, the differences in income become much larger. After discussing the equilibrium in the marriage market below, we address the links between family income distribution and efficiency in our equilibria.

What about the marriage market? Compatible, unemployed single people encounter each other at rate $\mu_k, k = m, w$. Denote $\hat{V}_m(p)$ the value for single men with productivity p of searching in the marriage market. Obviously, for this man there is a reservation value, call it $R_m(p)$, such that he is not willing to marry a woman, even if she is compatible, with productivity lower than $R_m(p)$. For a woman with productivity p we can define $\hat{V}_m(p)$ analogously.

Then,

4 Please note, in class formation as in Burdett and Coles (2006), the correlation in the productivity of those who marry arises without assuming any correlation in the incomes of those who meet. If one were to assume a matching process in which people with similar income are more likely to meet, this would only strengthen those results.

$$r\hat{V}_m(p) = \mu_m \int_{R_m(p)}^{R_w^{-1}(p)} V_{0,0}\left(p_H = \max\{p,\pi\}, p_L = \min\{p,\pi\}\right) dF_w(\pi),$$

3

$$r\hat{V}_w(p) = \mu_w \int_{R_w(p)}^{R_m^{-1}(p)} V_{0,0}\left(p_H = \max\{p,\pi\}, p_L = \min\{p,\pi\}\right) dF_m(\pi).$$

The bounds in the integral simply imply that a single person of gender *k* with productivity *p* would not accept a marriage proposal from somebody with $\pi \leq R(p)$, nor get one from somebody with $\pi \geq R_{\sim k}^{-1}(p)$.

Definition 5. Equilibrium in the marriage market is a pair of value function $\hat{V}_m(p), \hat{V}_w(p)$ and reservation strategies $R_m(p), R_w(p)$ such that 3 holds for all *p*.

Of course, because all agents would rank any two (suitable) marriage candidates in the same order, we know from Burdett and Coles (2006) that, in any equilibrium for the marriage market the population will be assorted in classes, where the men in the top class marry women of the top class, men in the second class marry women in the second class, and so on, with the possibility that, for some parameter values, some men or some women with very low productivity may never find someone who would take them.[5]

Lemma 6 (Burdett-Coles). There is a unique equilibrium of a marriage market, which takes the form of a partition of $\left[\underline{p}, \overline{p}\right]$ into sets S_i^m for the population of men, and S_i^w for the population of women where $S_1^k = \left[R_k(\overline{p}), \overline{p}\right]$, $\ldots S_i^k = \left[R_k o^i R_k(\overline{p}), R_k o^{i-1} R_k(\overline{p})\right]$, $k \in \{w, m\}$, where all agents of gender *k* with productivity $p \in S_i^k$ always marry the first compatible member of $S_i^{\sim k}$ that they encounter.

If the number of sets S_i^m for men is *n* then the number of sets S_i^n for women will be $n-1$ (the least productive men never marry),

---

[5] It is an unfortunate feature of the model that these men or women that never marry also never work. This comes from our choice of sequence (first the marriage market, only then the labor market). We have explored the alternative where agents enter both markets simultaneously, but in this case the number of states to keep track of expands significantly, and the flavor of the results does not change. Hence, we opted for simplicity in this regard.

n (everybody marries eventually) or $n+1$ (the least productive women never marry).

If μ/r is very low, $n=1$. Also, n increases with μ/r and $n \to \infty$ as $\mu/r \to \infty$.

The link between inequality across agents and inequality across couples is behind an inefficiency in equilibrium that we derive and that reduces welfare. In this regard, please note that as frictions disappear in the marriage market, then everybody marries her or his equal and all couples lie along the 45° line. In couples less productive than p^* this means that one relatively unproductive worker, but slightly more productive than their spouse, remains in the labor market; couples more productive than p^* are left –by choice– without the benefits of home production. Without a subsequent labor market, this would be welfare enhancing. In our framework, this is not necessarily the case. A social planner would try to generate a negative correlation between the productivity of spouses, to ensure that the less productive workers in society are as often as possible the less productive worker in their marriages, hence facilitating that they stay at home and specialize in home production, while the most productive workers in society are also the most productive workers in their couple, facilitating that they stay in the market. The equilibrium, by generating a positive correlation across spouses productivities, would keep out of the labor market some highly productive agents (because they married even more productive spouses), and would keep in the labor market some very unproductive agent (because they married even less productive spouses).[6]

[6] A simple example is one where the population is divided in two halfs, with productivities p_1 and p_2, where p_2/p_1 is a very high number. If μ and α are both high enough, in equilibrium the p_1 agents only marry each other, the p_2 agents only marry each other, and the labor force will be composed of half the population, of which again half would be p_1 and half would be p_2. In this case, the more productive half of society would enjoy utility $(1+\alpha)p_2$ and the other half would enjoy $(1+\alpha)p_1$. A social planner would prefer it if each p_1 married a p_2 (and viceversa),

Corollary 7. The assortative nature of the marriage market equilibrium leads to an inefficient allocation in the labor market. In particular, some relatively productive individuals will stay at home if their spouse is even more productive, and some relatively unproductive individuals will stay in the labor market if their spouse is even less productive. An efficient outcome would require a negative correlation between the spouses productivities, so for every very productive man or woman there would be incentives to be always in the job market, married to a very unproductive spouse that stays always at home.

Notice that one's productivity at home is proportional to the productivity at work of one's spouse.

Corollary 8. If men and women are very similar (that is, both genders have similar population sizes and similar distributions F_k), then for large levels of μ / r, almost all agents have very similar productivity to their spouse, and thus most couples belong to the sets where, in equilibrium, ϕ = (1, 1, 1, 1) or ϕ = (1, 1, 0, 0). As we consider lower levels of μ / r, and the sets S_i are less numerous but larger (or, alternatively, if we allow for disparities between the population of men and the population of women) there are sometimes bigger productivity differences across spouses, and an increasing fraction of the couples population share the home burdens asymmetrically $\phi = (1,1,0,1)$ and $\phi = (1,0,0,\cdot)$.

Corollary 9. The standard deviation of household per-member incomes is larger than the standard deviation of individual productivities, both because highly productive individuals marry each other, and because those couples have a higher average participation rate than other couples.

---

ensuring in that case that all the p_1 agents stay at home and all the p_2 agents work, which yields the higher utility $(1+\alpha)p_2$ for all agents. Hence, the same sorting mechanism that makes income distribution more skewed among couples than among individuals also leads to a loss of expected utility for all agents. Also, efficient sorting is likelier to emerge when productivity across agents is less variable.

Corollary 10. If there are asymmetries in the distribution of productivities of men and women, $F_w \neq F_m$ (say, because the here-unmodelled opportunities for education are not equal), in general the less productive gender will have a lower participation rate.

Corollary 11. If there are differences in population size between men and women, $\Omega_m \neq \Omega_w$, everything else being symmetric, the gender with the higher population will be less selective about marriage partners (have a lower $R_k(p)$), have a higher average labor-participation rate (since many of them will marry partners of the opposite sex that are less productive, since they are less selective), marry faster, and be likelier to have a low-class of individuals that never marry.

4. CONCLUSIONS

We have developed a model where the choice of marriage partner is endogenous, and once the couple is formed, it jointly decides its labor supply and home production. We find that the equilibrium involves different labor search strategies for different couples, and that often married agents – even the more productive spouse within the household, or somebody who has relatively high productivity among the population – stay at home. Couples of spouses with similar productivities to each other tend to choose strategies where both spouses do the same thing, while asymmetric couples tend to have asymmetric strategies. The latter kinds of couples tend, in equilibrium, to be less abundant (due to the assortative nature of equilibria), and more so as the technology for meeting potential spouses improves.

We find that the results we underscore in the corollaries in Section 3 match a number of findings in the empirical literature. Besides the facts mentioned in the Introduction, the findings about who marries whom tend to reconcile the results in Schwartz and Mare (2005), but the implications about income inequality do not necessarily follow, since in any equilibria where the two spouses in the couple behave symmetrically, in about half the households at any given time the less productive spouse is in the market and the more productive one stays at home. This means the income distribution among households may be more unequal than the productivity distribution among individuals. Thus, the results in Cancian et al. (1993) are also consistent with our theoretical results.

5. APPENDIX

5.1 Proof of Proposition 1

We apply the same procedure that we used in the text for the strategy $\phi=(1,1,1,1)$ now to the other three candidate strategies (not ruled out by Lemma 1): $\phi=(1,1,0,0)$, $\phi=(1,1,0,1)$, and $\phi=(1,0,0,\cdot)$.

Consider first $\phi=(1,1,0,0)$. In this case, the value functions become

$$V_{00}=\frac{(1+\alpha)\lambda(p_L+p_H)+(r+2\lambda+\delta)h}{r(\delta+2\lambda+r)},$$

$$V_{01}=\frac{\delta\lambda(1+\alpha)(p_L+p_H)+(1+\alpha)r(\delta+2\lambda+r)p_L+(r+\delta)(\delta+2\lambda+r)h}{r(\delta+r)(\delta+2\lambda+r)},$$

$$V_{10}=\frac{\delta\lambda(1+\alpha)(p_L+p_H)+(1+\alpha)r(\delta+2\lambda+r)p_H+(r+\delta)(\delta+2\lambda+r)h}{r(\delta+r)(\delta+2\lambda+r)},$$

$$V_{11}=\frac{\left[r^2+2(1+\alpha)\delta\lambda+r(2\lambda+(2+\alpha)\delta)\right](p_L+p_H)+2\delta(\delta+2\lambda+r)h}{r(2\delta+r)(\delta+2\lambda+r)},$$

and the incentive compatibility conditions require only $V_{01}>V_{00}$ and $V_{10}\geq V_{11}$, since the latter makes $V_{01}\geq V_{11}$ redundant. This narrows to

$$p_L>\ g_2(p_H)\equiv\frac{\lambda p_H}{\delta+\lambda+r}$$

$$p_L\geq g_3(p_H)$$
$$\equiv\frac{\left(\left[r^2+r(\alpha\delta+3\delta+2\lambda)+\delta(\alpha\delta+2\delta+\alpha\lambda+3\lambda)\right]\right)p_H-(\delta+r)(\delta+2\lambda+r)h}{\delta\alpha(\delta+3\lambda)+\delta\lambda+\alpha r^2+2\alpha r(\delta+\lambda)},$$

where we know that $g_3(p_H) < p_H$ only when $p_H < p^*$, as defined above, and that $g_2 \geq g_3$ if

$$p_H < p^\circ \equiv \frac{h(\delta+\lambda+r)}{\delta(\alpha+2)+(1-\alpha)\lambda+r}.$$

Therefore, the region where $\phi=(1,1,0,0)$ is an optimal strategy, is the one above g_2 for $p_H < p^\circ$, and above g_3 for $p_H \in \left[p^\circ, p^*\right]$. Consider now the job search strategy $\phi=(1,0,0,\cdot)$. Under this strategy,

$$V_{00} = \frac{\lambda(1+\alpha)p_H + (\delta+\lambda+r)h}{r(\delta+\lambda+r)}$$

$$V_{01} = \frac{\lambda\delta(1+\alpha)p_H + r(1+\alpha)(\delta+\lambda+r)p_L + (r+\delta)(\delta+\lambda+r)h}{r(\delta+r)(\delta+\lambda+r)}$$

$$V_{10} = \frac{(1+\alpha)(\lambda+r)p_H + (\delta+\lambda+r)h}{r(\delta+\lambda+r)}$$

$$V_{11} = -\frac{r(\delta+\lambda+r)(r+2\delta+\alpha\delta)(p_L+p_H)+2(1+\alpha)\delta^2\lambda p_H + 2\delta(\delta+r)(\delta+\lambda+r)h}{r(\delta+\lambda+r)(\delta+r)(2\delta+r)},$$

and optimality requires $V_{01} \leq V_{00}$ and $V_{11} \leq V_{10}$. The former translates into $p_L \leq g_2(p_H)$; the latter translates into

$$p_L \leq g_4(p_H) \equiv \frac{\lambda(h+\alpha p_H)}{(r+2\delta)(1+\alpha)+\lambda}.$$

As it turns out, g_2 is the binding upper bound when $p_H \leq p^\circ$, and viceversa.

To conclude, consider now the job-search strategy $\phi=(1,1,0,1)$. The value functions are straightforward to obtain yet rather messy, so we skip directly to the incentive compatibility conditions, which require simply $V_{10} \geq V_{11} > V_{01} > V_{00}$.

From the solutions of the value functions we derive that $V_{01} > V_{00}$ corresponds to $p_L > g_4(p_H)$. Meanwhile, $V_{10} \geq V_{11}$ holds if and only if $p_L \leq g_1(p_H)$, and $V_{11} > V_{01}$ if and only if $p_L < g_3(p_H)$. Since we know that the former is the binding constraint if $p_H > p^*$, and viceversa, we conclude that the couples for whom $\phi = (1,1,0,1)$ is the best job-search strategy are those that satisfy

$$p_H > p^* \text{ and } g_1(p_H) \geq p_L \geq g_4(p_H) \text{ or}$$
$$p^* \geq p_H \geq p^\circ \text{ and } g_3(p_H) \geq p_L \geq g_4(p_H).$$

5.2 Link to a Direct Utility Function

Here we address the link between the indirect utility function we use, and the direct function more commonly used in the literature, with features that include a fixed amount of available time that can be used either to work, as an input in home production, or to consume leisure; and include a home production function that uses time from the individuals and goods produced as inputs.

In the model, individuals either participate in the labor market full time or not at all. Income derived from the labor market for a couple, the independent variable in this utility function, has a domain that is not a dense interval in the real line, but a set of four discrete points (the income he can get, the income she can get, the income can get together, and 0 –the income that they get if neither works). Label these four points to be $y_0 = 0$, $y_1 = p_L$, $y_2 = p_H$ and $y_3 = p_L + p_H$, and consider the choice of how to allocate the resulting non-working time between leisure and home production. The option of allocating 0 hours to home production yields utility u_i to a couple with market income y_i, and the optimal allocation between leisure and home production yields utility $v_i \geq u_i \geq y_i$. Our only other restriction relative to some papers in the literature is that we assume that an agent working full time in the market cannot work at all at home. Hence, $h \equiv v_0$ follows with no loss of generality, $v_3 = y_3$ follows from this restriction, and we can define the values α_1 and α_2 by the equation $v_i = y_i(1+\alpha_i) + v_0$.

Consider a general utility $u(l, c, d)$ as a function of leisure l, consumption of market goods c, and consumption of domestic goods d. Commonly, we assume that d is an increasing function of homework hours $d = f_1(h_h)$, c an increasing function of market work hours and productivity $c = pf_2(h_m)$, and l is the time left after working in both, $l = H - h_h - h_m$. We are imposing, as we said above, restrictions on this general problem. The first is that $h_m \in \{0, H\}$, which implies that for each spouse there is a binary choice: either $h_h = 0$, $h_m = H$ and contribute $p_i f_2(H)$ to total c, or $h_m = 0$ and contribute 0 to total c. There are four types of household, defined by this binary choice. In type 0, $h_{Lm} = h_{Hm} = 0$, $c = 0$, and the couple maximizes $u(l, 0, f_1(2H{-}l))$, choosing an optional value l_0 that satisfies the first order condition $u_1() = u_3() f_1()$, that yields utility $u_0 = u(l_0, 0, f_1(2H{-}l_0))$. In type 1, where L works, $h_{Lm} = H$, $h_{Hm} = 0$, $c = p_L\Gamma$ (where $\Gamma = f_2(H)$ is a constant), and couples maximize $u(l, \Gamma p_L, f_1(H{-}l))$, again choosing an optimal value l_1 that satisfies analogous FOCs and implicitly yields utility $u_1 = u(l, \Gamma p_L, f_1(H{-}l_1))$. In type 2, where H works, obviously the couple is optimizing $u(l, \Gamma p_H, f_1(H{-}l))$ and deriving $u_2 = u(l_2, \Gamma p_H, f_1(H{-}l_2))$. In type 3, where both spouses work, $h_{Lm} = h_{Hm} = H$, $c = (p_H + p_L)\Gamma$, $l = 0$ and the derived utility is going to be $u_3 = u(0, \Gamma(p_L + p_H), f_1(0))$. At this point, these FOCs and implicit values u_i are obtained with no loss of generality across the set of possible functions $u(l, c, d)$, applying as a unique restriction relative to some literature that $h_m \in \{0, H\}$. It is natural (and again only a normalization) to assume that $f_1(0) = 0$, $u(0,y,0) = y$. Again with no loss of generality define the parameter that we called h in the paper as the value u_0, and derive that $u_3 = (p_L + p_H)\Gamma$. The only restriction we are imposing here is that $u(l_1, \Gamma p_L, f_1(H{-}l_1))/u(H, \Gamma p_L, 0) = u(l_2, \Gamma p_H, f_1(H{-}l_2))/u(H, \Gamma p_H, 0)$, which is guaranteed, among others, by any u function that his homogeneous in its first and third components.

With this notation, the only three restrictions we are imposing on the most general utility functions common in the literature are: *a)* That individuals either participate in the labor market or not at all. *b)* That individuals who work in the market do not work at home. *c)* That $\alpha_1 = \alpha_2$.

Restriction *a)*, as mentioned before, is something done quite frequently in broader literatures since Hansen (1984), and done in this literature, for instance, in Rogerson and Wallenius. Restriction *b)* is not too stringent. Only restriction *c)* is a loss of generality relative to the literature, and we do not feel it is big here. Again, in the absence of restriction *a)*, if the range of the possible incomes was an interval of the real line, restriction *c)* would amount to choosing a very specific functional form for utility (one where utility is homogeneous in leisure and home production, including but not limited to Cobb-Douglas). But given the granularity of the range, restriction *c)* is not that stringent. A discrete domain implies that order, not curvature, is the relevant attribute.

References

Bonilla, R., and F. Kiraly (2013), "Marriage Wage Premium in a Search Equilibrium," *Labour Economics*, 2, pp. 107-115.

Bonilla, R., F. Kiraly, and J. Wildman (2015), *Beauty Premium and Marriage Premium in Search Equilibrium: Theory and Empirical Test*, CESifo Working Paper, No. 5242.

Boulier, B., and M. Rosenzweig (1984), "Schooling, Search, and Spouse Selection: Testing Economic Theories of Marriage and Household Behavior," *Journal of Political Economy*, Vol. 92, No. 4, pp. 712-732.

Brien, M., Lee Lillard, and S. Stern (2006), "Cohabitation, Marriage, and Divorce in a Model of Match Quality," *International Economic Review*, Vol. 47, pp. 451-494.

Burdett, K., and M. Coles (2006), "Marriage and Class", *The Quarterly Journal of Economics*, Vol. 112, pp. 141-168.

Cancian, M., S. Dazinger, and P. Gottschalk (1993), "Working Wives and Family Income Inequality among Couples," in S. Dazinger and P. Gottschalk (eds.), *Uneven Tides: Rising Inequality in America*, Russell Sage Foundation, pp. 195-221.

Jaquemet, N., and J. M. Robin (2013), *Assortative Matching and Search with Labor Supply and Home Production*, CEMMAP Working Paper.

Lovász, A., and Ágnes Szabó-Morvai (2014), *Does Subsidized Childcare Matter for Maternal Labour Supply? A Modified Regression Discontinuity Analysis*, Working Paper, HETFA Research Institute.

Masters, A. (2008), "Marriage, Commitment and Divorce in a Matching Model with Differential Aging," *Review of Economic Dynamics*, Vol. 11, pp. 614-628.

Powell, L. (1997), "The Impact of Child Care Costs on the Labour Supply of Married Mothers: Evidence from Canada," *The Canadian Journal of Economics*, Vol. 3, No. 3, pp. 577-594.

Rogerson, R., and J. Wallenius (2012), "Retirement, Home Production and Labor Supply Elasticities," *2012 Meeting Papers*, No. 1, Society for Economic Dynamics.

Rogerson, R., and J. Wallenius (2013), "Nonconvexities, Retirement, and the Elasticity of Labor Supply," *American Economic Review*, Vol. 103, No. 4, pp. 1445-1462.

Schwartz, C. (2010), "Earnings Inequality and the Changing Association between Spouse's Earnings," *American Journal of Sociology*, Vol. 115, No. 5, pp. 1524-1557.

Schwartz, C., and R. Mare (2005), "Trends in Educational Assortative Marriage, 1940-2003," *Demography*, Vol. 42, No. 4, pp. 621-646.

Violante, G., B. Guler, and F. Guvenen (2012), "Joint-Search Theory: New Opportunities and New Frictions," *Journal of Monetary Economics*, Vol. 59, pp. 352-369.