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Fiscal Moral Hazard Due to Monetary Integration

Abstract

We develop a model in which a single currency plays the role of medium of exchange in two countries, while their governments are free to determine their fiscal balance and the extent to which they need to extract seigniorage from the common currency. We show that the actions of each government affect the economic performance of the other country, due to their trade relationship and, mostly, due to their monetary integration. We then endogenize each government's fiscal policy, and find that in equilibrium they will choose higher deficits than if they did not share a currency. Moreover, their policy choices are inefficient in the sense that if they could negotiate and commit their fiscal policy, they would choose smaller deficits. The inefficiency is worst if one of the partners is very small, or very unproductive, relative to the other, as the moral hazard on the smaller or poorer government would be larger.

Resumen

Elaboramos un modelo en el cual una moneda única desempeña el papel de medio de cambio en dos países, mientras

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que sus gobiernos son libres de determinar su saldo fiscal y la medida en la que necesitan obtener señoreaje de la moneda común. Demostramos que las acciones de cada gobierno afectan el desempeño económico del otro país, debido a su relación comercial y, fundamentalmente, debido a su integración monetaria. Entonces, endogenizamos la política fiscal de cada uno de los gobiernos, y hallamos que en equilibrio elegirán mayores déficits que si no compartieran una moneda. Además, sus opciones de política son ineficientes en el sentido de que si pudieran negociar y comprometer su política fiscal, elegirían déficits menores. Su ineficiencia es peor si uno de los socios es muy pequeño o muy improductivo en comparación con el otro, ya que el riesgo moral para el gobierno más pequeño y más pobre sería mayor.

1. INTRODUCTION

Sharing a currency can create a strong and interesting link among two countries. One can argue, for instance, that such an arrangement facilitates trade, by reducing both the transaction costs (including exchanging one currency for another, or keeping positive balances in several monies) and the risks (mostly, from the volatility of the exchange rate) associated with international commerce. In some cases where countries have chosen to do away with their national currency (like the dollarization of El Salvador and Ecuador, or the creation of the euro), these arguments have shown to be particularly relevant, as their economies had become more integrated to the world, and interest rates (which reflect exchange rate risk) fallen, upon the change in currency. For a discussion, see Trejos (2003).

Once currencies are shared, on the other hand, rules matter. In cases where one nation simply starts using as its medium of exchange the money of another nation, two problems emerge: that the adopting country loses control of monetary policy –and the economic cycle in the adopting country may be very unsynchronized with that of the issuing country, so the resulting policy is particularly ill adapted to the latter– and that not having a currency of ones own implies that ones government does not extract any seigniorage –and local citizens are still *taxed* by the money creation of the issuer.

At least during the creation of the euro, the point was made that a group of members in an economic union, with coordinated policies, each having a say in the monetary policy decisions, and acting as a co-issuer of the currency, could yield the benefits pointed out in the first paragraph without the sacrifices pointed out in the second. But this decision, however, does carry its own costs and risks. One may worry, most importantly, that there may be moral hazard regarding fiscal issues (since *my* fiscal imbalance will partly be paid by extracting seigniorage from *our* currency, and among other things this increases *your* inflation). In other words, that a common currency and monetary policy would tempt the member governments into fiscal laxity, with its eventual consequences.[1]

This trade-off between the trade facilitation brought about by currency sharing, and the failures of macroeconomic policy in the absence of perfect coordination when a currency is shared, is clearly at the heart of several important issues of our time, and notably in the propagation of the fiscal crisis across European Union members. A little bit of theory can help the discussion.

We approach this question with a model where money is essential, in the sense that its use emerges endogenously from the frictions in the exchange process. This type of model can

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1 In the early history of the United States, some thought that this moral hazard problem could destroy the Union, and they chose not to have a single, government-issued federal currency for a century. It was only when the constitutional conditions emerged, forcing states into binding constraints about their public finances, that a federal *dollar* was created. Similarly, in the United Kingdom and Denmark, the discussions that eventually led to them not joining the euro included invariably that, as relatively rich members of the single currency, they would be forced by the circumstances to transfer resources to the unavoidable fiscal problems of their smaller neighbors.

be used to, among other things, study the forces that determine endogenously which currency circulates where. Also, we believe that the strategic interaction among fiscal authorities, brought about by the common currency, should be the focus of study. In this paper, we develop a theoretical model that does precisely that. It is a model of search and matching with a double coincidence of wants problem –so that the liquidity of an intrinsically useless asset that serves as medium of exchange is the natural result of the environment. It is a model where trade with foreigners is comparatively less frequent than among locals, but not impossible, so that the question of which currency circulates where, and who buys from whom can be posed. It is also a model where local governments can extract seigniorage –generate a revenue flow by reducing the value of money– as part of their public finances.

The basic structure of the model is inspired by Matsuyama et al. (1993). Following Trejos and Wright (1995) and Shi (1995) we change the MKM model by introducing a bargaining game that makes prices (though not nominal exchange rates) endogenous, along the lines of Trejos and Wright (2001). In these models, each country issues its own currency, as the key question has to do with *spontaneous dollarization*: obtaining equilibrium in which one currency (say, the peso) only circulates in the country that issues it while another currency (say, the dollar) circulates everywhere, as a consequence of the private choices of individuals, and not by policy design. From that model, one can also predict that another equilibrium, where every currency circulates everywhere, exists and is particularly robust, and in it the different monies become perfect substitutes in a relevant way.

In this paper, building upon that last finding, we assume that there is a single currency issued by a joint central bank, as we want to focus on situations where the same money circulates everywhere, and both countries coordinate to determine the real money supply, but act independently in their choice of fiscal policy (that is partly financed by seigniorage).

We find that sharing a currency creates among the two governments a miscoordination problem akin to moral hazard. The real value of money in *both* countries is affected by the fiscal responsibility (or lack thereoff) of *both* governments, and not surprisingly each one makes its choices thinking about its own citizens, with disregard of the effect they have on each other. In the end, in equilibrium, the chosen fiscal burden to pass to the union is too big. The policy objectives of both governments would be better achieved if there was a binding commitment device that limited their choices but, in the absence of such a binding device, they choose to extract more seigniorage, and the resulting equilibrium is Pareto inefficient. Monetary unions bring about fiscal troubles.

The bigger the asymmetry of size or productivity of both economies, the stronger the moral hazard incentives on the smaller economy, and thus the larger the deficit it chooses to run, and the costs that it passes on to its currency-union partner. In fact, there is a critical value of the size asymmetry and, beyond that, fiscal crisis is simply unavoidable.

We find these results from the theory very telling in explaining the mistakes in the design of the euro that led to the current crisis. We illustrate in the conclusion some empirical results that document the relationship between euro membership and fiscal laxity.

Section 2 describes the economic environment and Section 3 describes the equilibrium and the key results regarding the existence and uniqueness of equilibrium for the private economy, given the policy parameters. In Section 4, we endogenize those policy parameters, and work out the equilibrium choices of the governments. Section 5 discusses some relevant extensions and concludes.

2. ENVIRONMENT

Time is continuous and continues forever. There are two groups, or nationalities, with shares $n_1 = n$ and $n_2 = 1 - n$ of the total population. With no loss of generality, we assume $n \geq 1/2$

. Both populations grow at an exogenous rate $\gamma > 0$. All agents produce and consume goods, that come in many varieties, and which are not storable. A given agent always produces the same variety but changes over time which varieties she wishes to consume. The number and configuration of varieties imply that the frequency of self-production (the situation where an agent happens to want the variety he is able to produce) or of double coincidence of wants (the situation where each of two agents happens to produce the variety the other one wants) is zero. An agent's production variety, endowment and nationality are always observable.

The consumption of Q units of the right variety can deliver utility $u(Q)$, where $u(0)=0$, $u'(Q)>0$ and $u''(Q)<0$. The production of those Q units of the good requires a labor effort disutility $c(Q)=Q$. There is a value $\overline{Q}$ that satisfies $\overline{Q}=u\left(\overline{Q}\right)$.

Agents meet randomly, through a Poisson process. Someone from nationality i encounters other i-nationals (with whom he can suitably trade) at an arrival rate $\alpha_{ii}=\alpha$, and foreign or k-nationals ($k \neq i$) with arrival rate $\alpha_{ik}=\alpha\phi n_k / n_i$, where $\alpha > 0$ and $\phi \in [0,1)$. The parameter ϕ can be interpreted as the degree of integration between the two economies: when $\phi = 0$, there is no trade among citizens of different nationalities; when $\phi = 1$, a buyer is just as likely to encounter a member from a set of local sellers as to encounter a member from a set of the same measure of foreign sellers. There are no multi-agent meetings or centralized interactions of any kind; in particular, there is no Walrasian market where the entire population can exchange, at once and anonymously, at a market-clearing price.[2]

Because double coincidence of wants and self-production are impossible, and goods cannot be used as commodity money

[2] Note that this specification of the arrival rates means that domestic transactions are equally easy to come by in both domestic economies, that opportunities of domestic exchange are relatively easier to come buy than opportunities of foreign exchange, and that international trades are not equally important for both countries (they are more frequent from the point of view of the citizen from the less-populated country 2).

because they are not storable, the only way for agents to trade in this environment is if there exists an object that could be used as a medium of exchange. We assume that there is a central bank, common to both countries, that puts in circulation such an object, which we call money. Money is intrinsically worthless, and cannot be produced or consumed by a regular agent, but is storable and tradable. For simplicity we also assume it is indivisible, and cannot be held in more than one unit at a time.[3] The central bank puts the money in the market, by endowing a fraction M of newborn agents with one unit of it.

The existence of a monetary equilibrium would depend on expectations. In particular, if all agents expect that money is worthless in exchange, this expectation is self fulfilling. On the other hand, if they expect others to be willing to produce some amount Q of goods in exchange for money, they may be willing to produce themselves some amount q in exchange for money as well, and it is possible that a monetary equilibrium where money will have value would exist, if there was a fixed point where $q = Q > 0$. It is such equilibrium that we care about here.[4] We assume that Q is determined by bargaining. To be precise, if a buyer and a suitable seller expect a non-negative

[3] A more complicated model where money is divisible and accumulable could be built here, following the developments in Lagos and Wright (2005). But for the specific and very applied purposes of this paper the complexities of such generalization are not necessary. For a further analysis of the implications of indivisibility and its applications in monetary economics and in finance, see Trejos and Wright (2012).

[4] In Matsuyama et al. (1993) and Trejos and Wright (2001), each country was assumed to issue its own currency, as the main interest was on determining endogenously which currencies would circulate where, and whether an equilibrium with international currency (that is, where one money circulated only at home while another circulated at home and abroad) could emerge. In that model, a global equilibrium –particularly robust– always existed, where both monies circulated everywhere and were perfect substitutes. Here we cut to the chase and assume there is only one money, thus circumscribing the analysis to this last equilibrium.

surplus from exchange they enter a game of alternating offers, a-la-Rubinstein (1982), where the bargaining power of the seller is denoted σ. It is well known that this game equilibrium is a Q level that satisfies an axiomatic Nash solution, which be derived explicitly and corresponds to the formula used below.

In addition to the central bank, there are also two national governments, who extract a flow of taxation (or seigniorage) by taking away a part of the value of monetary transactions. We simply assume that the government i taxes away some of the goods produced by sellers from country i. To be precise, if a buyer and a seller meet and find exchange is possible and desirable, they bargain, the seller produces, and trades the produced goods for the buyer's s money. It is then that government from the seller's country may show up and, with probability μ_i confiscate the goods.[5] The fraction of the population of country i that holds cash at any given point in time is denoted m_i.

We define V_i as the discounted flow value of an agent from country i at a time when he is holding currency, V_{i0} his value when he has no cash. We call ν_{ik} the probability that an i-buyer agrees to trade with a k-seller of the right type when they encounter in the decentralized market, and Q_{ik} as the amount of output traded in that exchange.

There is a rationale for the actions of two national governments. We will consider two alternative forms of behavior for national government i: that it chooses μ_i trying to maximize the seigniorage collected $S_i = \mu_i[\alpha_{ii}m_i + \alpha_{ik}m_k](1-m_i)Q_i$, or that it does it by trying to maximize national welfare $W_i = m_iV_i + (1-m_i)V_{i0} + \omega S_i$, where $\omega > 0$ implies that the use of goods by government can contribute to general welfare.

[5] This approach is slightly different from the followed by Li (1995), who assumes the government encounters buyers according with some stochastic process, confiscates money, and send buyers to the production stage without consumption.

3. EQUILIBRIUM

The relevant Bellman equations here are

1 $$rV_i = \alpha_{ii}(1-m_i)\nu_{ii}\left[(1-\mu_i)u(Q_{ii})+V_{i0}-V_i\right]+ \\ +\alpha_{ik}(1-m_k)\nu_{ik}[(1-\mu_k)u(Q_{ik})+V_{i0}-V_i]$$

$$rV_{i0} = \alpha_{ii}m_i\nu_{ii}\left[V_i-V_{i0}-Q_{ii}\right]+\alpha_{ik}m_k\nu_{ik}\left[V_i-V_{i0}-Q_{ki}\right].$$

The LHS of the first equation is the flow value of being a buyer from country i, where r is the rate of time preference, equal on the first term of the RHS to the arrival rate of local producers of the variety one wants α_{ii}, times the probability $1-m_i$ they hold no money and may be willing to produce, times the probability ν_{ii} that both find this trade satisfactory, times the surplus involved in the exchange: the shift in value from buyer to seller $V_{i0}-V_i$ plus the utility enjoyed in consumption $u(Q_{ii})$, provided the government did not tax the goods before that, $1-\mu_i$. The second term of the RHS is analogous and corresponds to the payoff from meeting foreign sellers. The other Bellman equations are interpreted in a similar manner.

A steady state, according to the law of motion of the distribution of money holdings, requires

2 $$\dot{m}_i = \alpha_{ik}(1-m_i)m_k\nu_{ik}+\alpha_{ik}m_i(1-m_k)\nu_{ik}+\gamma(M-m_i)=0.$$

Following Rubinstein (1982), the amounts Q_{ik} traded in an exchange between a buyer from i and a seller from k satisfy the axiomatic Nash bargaining solution

3 $$Q_{ik} = \arg\max_q\left[V_{i0}-V_i+(1-\mu_k)u(q)\right]^{1-\sigma}\left[V_k-V_{k0}-q\right]^{\sigma},$$

and where σ is the bargaining power of the seller.

In turn, buyers' trading strategies, taking into account the non-negativity of the factors in 3 require

$$\nu_{ik} = \begin{cases} 1 & \iff & u^{-1}(V_i - V_{i0}) & \geq & (V_k - V_{k0})/(1-\mu_k) \\ 0 & & otherwise & & \end{cases}. \tag{4}$$

A stationary monetary equilibrium is a collection of exchange quantities, money holdings and trading strategies $\{V_i, V_{i0}, Q_{ik}, m_i, \nu_{ik}\}$ that satisfies equations 1, 2, 3 and 4, with $Q_{ik} > 0$ at least for some i,k, taking as given the policy parameters μ_i, M.

Because the set of equilibrium is potentially very large, and because we are primarily interested in questions that arise in a situation where money truly circulates everywhere, we will focus on what we will call Full Circulation Equilibrium (FCE), that is, stationary monetary equilibrium where $\nu_{ik} = 1 \; \forall \; i,k$.[6] We will further simplify the analysis by giving buyers all the bargaining power, so $\sigma = 0$, which significantly cuts the number of endogenous variables and the complexity of the algebra involved, without changing too significantly the economics of the problem.

The assumption that buyers have all the bargaining power implies that there is no value in being a seller ($V_{i0} = 0$), that a seller from a given country always sells at the same price regarding of the nationality of the buyer ($Q_{ii} = Q_{ki} \equiv Q_i$) and that $V_i = Q_i$. Furthermore, in this simple setup the solution to the steady state conditions 2 is simply $m_1 = m_2 = M$. Hence, in a full-circulation equilibrium the Bellman equations (1)

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[6] Readers familiar with the search literature know that, if there is only one nationality in this model, there are always at least two equilibria: monetary, where $\nu = 1$, and degenerate where $\nu = 0$. With two nationalities it is possible that while all buyers trade with their countrymen sellers, the arrival rates of foreign trade, and of government confiscation, are different. If the difference is large enough, it is possible that buyers from a country where money is more valuable would rather wait for local seller, than spend their money on a foreigner that gives less for it because he values it less. Hence, it is possible that some ν_{ik} are 0 and others are , and there are many possible combinations that constitute equilibria.

and bargaining solution 3 are met provided that $\mathbf{Q} = (Q_1, Q_2)$ satisfies

5 $$\frac{rQ_1}{1-M} = \alpha_{11}\left[(1-\mu_1)u(Q_1) - Q_1\right] + \alpha_{12}\left[(1-\mu_2)u(Q_2) - Q_1\right]$$
$$\frac{rQ_2}{1-M} = \alpha_{21}\left[(1-\mu_1)u(Q_1) - Q_2\right] + \alpha_{22}\left[(1-\mu_2)u(Q_2) - Q_2\right].$$

From 4, it is easy to derive that the condition $v_{ik} = 1 \;\; \forall i,k$ is equivalent to the condition $\mathbf{Q} \in \Omega \equiv \left\{\left(Q_1, Q_2\right) \mid (1-\mu_1)u\left(Q_1\right) \geq Q_2, \left(1-\mu_2\right)u\left(Q_2\right) \geq Q_1\right\}$.

Hence, a FCE is simply a combination $\mathbf{Q} \in \Omega$ which satisfies 5. The following proposition establishes the existence and uniqueness of a FCE, for certain values of the policy parameters.

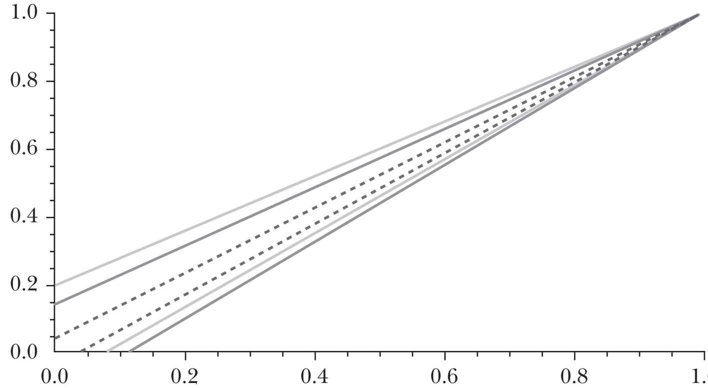
Proposition 1. For all $r > 0$, for low enough ϕ, μ_1, μ_2 there exists a FCE. If the equilibrium exists, it is unique.

Proof. For all Q_1 there is a unique value of Q_2, call it $Q_2 = \Psi(Q_1)$, that satisfies the first equation in 5, because any expression of the form $aQ_1 - bu(Q_1)$, with $a, b > 0$, is a first decreasing and then increasing, convex function of Q_1. By the same token, for all Q_2 there is a unique value $Q_1 = \Phi(Q_2)$ that satisfies the second equation in 5. Furthermore, by the Implicit Function Theorem, we know that Ψ and Φ are strictly increasing and strictly concave, and that $\Psi(0)$ and $\Phi(0)$ are both positive. This is sufficient to guarantee that there is a unique pair $(Q_1^*, Q_2^*) \in R_+^2$ such that $Q_1^* = \Phi(Q_2^*)$ and $Q_2^* = \Psi(Q_1^*)$, satisfying 5 and so a candidate for a FCE.

With $\phi = \mu_1 = \mu_2 = 0$, then, it is clear that $Q_1^* = Q_2^* \in (0, \overline{Q})$, and thus that $(Q_1^*, Q_2^*) \in \Omega$, a unique FCE. Consider now alternative values of μ_i, still under $\phi = 0$. Define now $\overline{\mu}_i(\mu_k)$ as the value of μ_i, given μ_k, under which $u(Q_i) = Q_k$. Verify that $\overline{\mu}_i(0) < 1$. and $\partial\overline{\mu}_i / \partial\mu_k > 0$. Therefore, $\mu_i \leq \overline{\mu}_i(\mu_k) \Longleftrightarrow u(Q_i) \geq Q_k$. Defining $\Theta \equiv \left\{\left(\mu_1, \mu_2\right) \mid \mu_1 \leq \overline{\mu}_2(\mu_1), \mu_2 \leq \overline{\mu}_1(\mu_2)\right\}$, and a FCE exists when $(\mu_1, \mu_2) \in \Theta$. One can notice that Θ is either the empty set, or a compact, closed set, with borders $\overline{\mu}_k(\mu_i)$ and containing the origin. Furthermore, at $\phi = 0$ we know that Θ is not

Figure 1

FULL CIRCULATION EQUILIBRIA



Note: FCE exists for (μ1, μ2) between dark gray curves (benchmark parameters), dotted (near autarky) or light gray (larger size asymmetry).

empty. Since all implicit functions in this problem are C^{∞} in ϕ, there is some positive value $\tilde{\phi}$ such that, if $\phi = \tilde{\phi}$, $\mu_1 = \mu_2 = 0$ then $u\left(Q_1^*\right) = Q_2^*$. Thus, $\Theta \cap \Omega \neq \varnothing \Longleftrightarrow \phi < \tilde{\phi}$. ■

Following the proposition, the set Θ of values of (μ_1, μ_2) under which the FCE exists has roughly a shape like the one shown in Figure 1.[7] As ϕ increases, the boundaries of Θ move towards the origin, reducing the size of the set Θ , which always

[7] Notice that the consequence of picking too high a value of μ_i would be to push Q_i so low that $k \neq i$ buyers no longer consider it worthwhile to purchase from i sellers, and prefer to wait for a fellow countryman instead. Then, the FCE does not exist, although it is not clear either that another form of monetary equilibrium could take its place. Why? Because if i sellers are no longer selling to k buyers, then money is only leaving, not entering, country i, and in steady state we would have less sellers (and less value) in k once it has to house all the money. It may be that the very high μ_i carries with it that assuming FCE implies i sellers dont sell to k buyers (contradicting FCE) and that assuming a different monetary equilibrium, with all the money in k , implies i sellers do sell to k buyers (contradicting this alternative equilibrium), so no pure-strategies monetary equilibrium exists in that case.

contains the origin. When ϕ reaches $\tilde{\phi}$, Θ collapses into the origin, and for higher levels of ϕ no FCE can exist for any policies of the local government. Of course, nothing guarantees mathematically that $\tilde{\phi} < 1$, although $\phi > 1$ would imply, nonsensically, that barriers to international trade are smaller than barriers to local trade.

It will also be convenient to know the following lemma, guaranteeing that the equilibrium Q^* values are decreasing on both confiscation rates μ, and that each countrys prices are more sensitive to the actions of its own local government than to the actions of the foreign government.

Lemma 2. $\partial Q_i^* / \partial \mu_j < 0$ for $i, j = 1,2$. Also, if $n \sim 1/2$ and $\mu_1 \sim \mu_2$, $\partial Q_i / \partial \mu_i > \partial Q_i / \partial \mu_k$ and also $\partial Q_i / \partial \mu_i > \partial Q_k / \partial \mu_i$.

Proof. It is a straightforward application of the Implicit Function Theorem on 5. It is easy to show that our Bellman rewrites into

6
$$\begin{aligned} n(1-\mu_1)u(Q_1) + \phi(1-n)(1-\mu_2)u(Q_2) - ZQ_1 = 0 \\ \phi n(1-\mu_1)u(Q_1) + (1-n)(1-\mu_2)u(Q_2) - YQ_2 = 0 \end{aligned}$$

where $Z = \frac{rn}{\alpha(1-M)} + n + \phi(1-n)$ and $Y = \frac{r(1-n)}{\alpha(1-M)} + (1-n) + \phi n$, which are constant and positive.

Applying the Implicit Function Theorem we obtain the following derivatives, all negative as expected

7
$$\begin{aligned} \frac{\partial Q_1}{\partial \mu_1} &= \frac{nu(Q_1)[(1-n)(1-\phi^2)(1-\mu_2)u'(Q_2) - Y]}{E} < 0 \\ \frac{\partial Q_1}{\partial \mu_2} &= -\frac{(1-n)\phi Y u(Q_2)}{E} < 0 \\ \frac{\partial Q_2}{\partial \mu_1} &= -\frac{n\phi Z u(Q_1)}{E} < 0 \\ \frac{\partial Q_2}{\partial \mu_2} &= \frac{(1-n)u(Q_2)[n(1-\phi^2)(1-\mu_1)u'(Q_1) - Z]}{E} < 0 \end{aligned}$$

where $E = n(1-\mu_1)u'(Q_1)\left[(1-n)(1-\phi^2)(1-\mu_2)u'(Q_2)-Y\right]$ $-\left[(1-n)(1-\mu_2)u'(Q_2)-Y\right]Z > 0.$ The sign of the first and last numerator can be derived from 6 and the concavity of $u(\bullet)$. ■

4. EQUILIBRIUM POLICY

Instead of taking the policy parameters μ_i as given, we now endogenize them, by considering the Nash equilibrium of a game in which the each local government i chooses μ_i as a best response to the choice μ_k of the counterpart. We will consider two scenarios that vary according to the objective function each government is pursuing: in the first case, we assume their goal is to maximize the seigniorage collected; in the second, it is to maximize the welfare of its own citizens. In the next subsection, this is done assuming that their choices are free and there is no way to make a binding commitment, so we look at the Nash equilibrium of a non-cooperative game. In the following one, we work out the bargain equilibrium in the cooperative game where binding agreement is possible, for the sake of comparison.

4.1 Non-cooperative Solution

We look first at the case where each local government i tries to maximize seigniorage. The first step is to derive the best response functions $\mu_1^S(\mu_2)$ and $\mu_2^S(\mu_1)$, taking M as given. Notice that if $\mu_i > \bar{\mu}_i(\mu_k)$ then i sellers will not be able to sell to k buyers, all money will leave i, and therefore no seigniorage is collected. This means that government i would always select $\mu_i^S(\mu_k) \leq \bar{\mu}_i(\mu_k)$, consistent with the existence of a FCE. Recall we are considering that local government can only expropriate goods from transactions carried over with national sellers, which along with the steady-state condition implies that:

8
$$\mu_i^S(\mu_k) = \underset{0<\mu_i<\bar{\mu}_i(\mu_k)}{\arg\max}\{D_i \mu_i Q_i\}$$

where $D_i = \alpha M(1-M)\left[1+\varphi\frac{n_k}{n_i}\right]$.

If we assume that buyers and sellers always trade, independent of 4, and simply worked out $S_i(\mu_i)$, we would notice it behaves as sort of Laffers Curve, that first increases and then decreases with μ_i (the decline produced by the adverse effect that a higher probability of confiscation has on local equilibrium quantities). It means there is always a $\hat{\mu}_i$ that maximizes $S_i(\mu_i)$, defined by $\hat{\mu}_i = -Q_i(\hat{\mu}_i)/Q_i'(\hat{\mu}_i)$. However, it may or may not be the case that $\hat{\mu}_i$ is consistent with FCE, which requires $\mu_i \leq \bar{\mu}_i(\mu_k)$. If this constraint is not binding then $\hat{\mu}_i$ corresponds to the best response value for government *i*. If it is, then the government, knowing that choosing $\hat{\mu}_i > \bar{\mu}_i(\mu_k)$ implies turning $D_i = 0$ and thus losing all revenue, would prefer the constrained best response $\mu_i = \bar{\mu}_i(\mu_k)$. Hence,

9
$$\mu_i^S(\mu_k) = \min\{\bar{\mu}(\mu_k), \hat{\mu}_i(\mu_k)\}$$

The best response of the government may be to raise its confiscation rate all the way up to the level where foreign buyers are indifferent between buying or selling from his citizens.

On the other hand, if government *i* is committed to maximize the welfare of its population, then given *M* and μ_k, the best response function in this case can be expressed as

10
$$\mu_i^W(\mu_k) = \arg\max\{W_i \equiv Q_i(M + \omega D_i \mu_i)\}.$$

Again, if μ_i^W is an interior solution it has to satisfy first order conditions, which implies

11
$$\mu_i^W(\mu_k) = Q_i \,/\, \frac{\partial Q_i}{\partial \mu_i} - M \,/\, \omega D_i.$$

The properties proven so far also guarantee the existence of a Nash equilibrium to the non-cooperative game.

Proposition 3. There exists a Nash equilibrium $\mu^S = (\mu_1^S, \mu_2^S)$ in the seignorage-maximization game, and a Nash equilibrium $\mu^W = (\mu_1^W, \mu_2^W)$ in the domestic-welfare-maximization game.

Proof. Observe that the space of strategies $S_i \equiv [0,1]$ for $i = 1, 2$ is trivially nonempty, convex and compact. Moreover, S_i and W_i are continuous with respect to $\mu \in [0,1]^2$, given that Q_i is continuously differentiable in both parameters; and are quasiconcave with respect to μ_i. All this implies that the simultaneous games conformed by $\{\{1,2\}, S_i, S_i\}$ and $\{\{1,2\}, S_i, W_i\}$ satisfy the assumptions in Nash (1950), and hence have a Nash equilibrium. ■

This equilibrium may be a corner solution (where one or both governments take μ to the maximum compatible with the global circulation of money) or a interior solution.

Lemma 4. The best response functions $\mu_1^S(\mu_2)$ and $\mu_2^S(\mu_1)$ intersect once and only once in the interior of $[0,1]^2$.

Proof. Clearly, by continuity, monotonicity and concavity, if the functions $\hat{\mu}_i$ intersect in $[0,1]^2$, this intersection is unique. Moreover, the functions $\bar{\mu}_i$ necessarily intersect once in that interval. Notice that the function μ_i^S is equal to $\hat{\mu}_i$ as long as $\hat{\mu}_i < \bar{\mu}_i$ and equal to $\bar{\mu}_i$ after the intersection. Hence, the functions μ_i^S must also intersect once, and only once. ■

The work so far permits to characterize the best response function of local governments to both policy objectives. In particular, it can be shown that $\partial\mu_i^S / \partial\mu_k, \partial\mu_i^W / \partial\mu_k > 0$, and $\partial^2\mu_i^S / \partial\mu_k^2, \partial^2\mu_i^W / \partial\mu_k^2 < 0$ for all i, which means that best response functions to either policy are strictly increasing and strictly concave. It is straightforward from 11 also that $\mu_i^W(\mu_k) \leq \hat{\mu}_i(\mu_k)$ and, since the same constraint is binding for both problems then $\mu_i^W(\mu_k) \leq \mu_i^S(\mu_k)$, that is, a government concerned with the welfare of its local citizens will never choose a lower real value of money than one concerned with seigniorage. The decentralized equilibrium of the seigniorage-maximization game involves over-taxation.

4.2 Cooperative Solution

We now work out the equilibrium solution in cases where the governments can enter binding commitments regarding their actions, and choose to cooperate and commit on policy. Again, we look at two cases: one where governments are helping each other maximize total seigniorage and another where they are concerned about global welfare.

In the former case, the optimization problem writes as

$$\mu^{CS} = \underset{0\leq\mu_i\leq\bar{\mu}_i(\mu_k)}{\arg\max}\left\{nQ_1D_1\mu_1 + \left(1-n\right)Q_2D_2\mu_2\right\}.$$

In an interior solution, the equilibrium with cooperation must satisfy

12
$$\mu_1^{CS} = \mu_1^S(\mu_2^{CS}) - \left[\frac{(1-n)}{n}\frac{\partial Q_2/\partial\mu_2}{\partial Q_1/\partial\mu_1}\frac{D_2}{D_1}\right]$$
$$\mu_2^{CS} = \mu_1^S(\mu_1^{CS}) - \left[\frac{n}{(1-n)}\frac{\partial Q_1/\partial\mu_1}{\partial Q_2/\partial\mu_2}\frac{D_1}{D_2}\right]$$

and it becomes fairly clear that $\mu_i^S > \mu_i^{CS}$, so indeed we obtain inefficiently high confiscation rates –or inefficiently low real value of money– as a consequence of the lack of commitment.

If both governments make a commitment to maximize global welfare, the optimal choice corresponds to:

13
$$\mu^{CW} = \underset{0\leq\mu_i\leq\bar{\mu}_i(\mu_k)}{\arg\max}\left\{nQ_1(M+\omega D_1\mu_1) + (1-n)Q_2(M+\omega D_2\mu_2)\right\}$$

leading to the first order conditions:[8]

14
$$\mu_1^{CW} = \mu_1^W(\mu_2^{CW}) - \left[\frac{(1-n)}{n}\frac{\partial Q_2/\partial\mu_2}{\partial Q_1/\partial\mu_1}\frac{M+\omega D_2\mu_2^{CW}}{\omega D_1}\right]$$

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[8] Taking μ_i to its maximum value $\bar{\mu}_i$ implies that welfare in country i becomes 0, so we know that at least one of the equilibrium policy parameters μ_i is going to be an interior solution.

$$\mu_2^{CW} = \mu_1^{W}(\mu_1^{CW}) - \left[\frac{n}{(1-n)} \frac{\partial Q_1 / \partial \mu_1}{\partial Q_2 / \partial \mu_2} \frac{M + \omega D_1 \mu_1^{CW}}{\omega D_2} \right].$$

Since $\mu_i(\cdot)$ is increasing and the second term on the RHS of equations 12 and 15 are negative, then we can conclude that $\mu_i^{CS} \leq \mu_i^{S}$, and $\mu_i^{CW} \leq \mu_i^{W}$. Additionally, if $M < \omega\left(\frac{1-n}{n} \frac{\partial Q_1/\partial \mu_1}{\partial Q_2/\partial \mu_2}\right)$, then $\mu_i^{CS}(\cdot) < \mu_i^{W}(\cdot)$.

We can compare the interior solutions in both cases, and observe that

15

$$\mu_1^{CS} = \mu_1^{CW}(\mu_2^{CW}) - \left[\frac{(1-n)}{n} \frac{\partial Q_2 / \partial \mu_2}{\partial Q_1 / \partial \mu_1} \frac{M + \omega D_2 \mu_2^{CW}}{\omega D_1} \right]$$

$$\mu_2^{CS} = \mu_1^{CW}(\mu_1^{CW}) - \left[\frac{n}{(1-n)} \frac{\partial Q_1 / \partial \mu_1}{\partial Q_2 / \partial \mu_2} \frac{M + \omega D_1 \mu_1^{CW}}{\omega D_2} \right]$$

which implies that $\mu_i^{CW} \leq \mu_i^{W} \leq \mu_i^{S}$, since $\mu_i(\cdot)$ is increasing and the second term on the RHS of 15 is negative.[9]

We can also derive the sensitivity of the Nash equilibrium to the parameters of the model. Interestingly, one can show that

$$\frac{\partial \mu_1^{S}(\cdot)}{\partial n} \overset{s.}{=} (1-n)^2 \left(1-\phi^2\right)\left(1-\mu_2\right)^2 u'\left(Q_2\right)^2 - \left(2(1-n)+\phi^2(2n-1)\right)\left(1-\mu_2\right)u'\left(Q_2\right)Y + Y^2$$

$$\frac{\partial \mu_2^{S}(\cdot)}{\partial n} \overset{s.}{=} n^2 \left(1-\phi^2\right)\left(1-\mu_1\right)^2 u'\left(Q_1\right)^2 + \left(2n\left(\phi^2-1\right)-\phi^2\right)\left(1-\mu_1\right)u'\left(Q_1\right)Z + Z^2$$

9 These weak inequalities must hold in all cases, and not only if the equilibrium is an interior solution, because it may be that the constraint $\mu_i \leq \overline{\mu}_i$ is binding for the seigniorage-maximization case and not the welfare-maximization case, but the opposite cannot be true.

Table 1

RELATIVE DEFICIT BEFORE/AFTER ENTERING THE EURO AREA: 1986-2011

Covariates	*(1)* *Euro: 1986-2011* Log(Def_Rel)	*(2)* *Euro: 10 years of entrance* Log(Def/GDP_Rel)
Log(Debt_Rel)	1.154[b]	
	(0.564)	
Log(Pop)	−0.461[a]	−0.357[a]
	(0.109)	(0.0863)
Log(Debt/GDP_Rel)		1.542[a]
		(0.429)
Constant	0.984[a]	0.481[b]
	(0.289)	(0.234)
Observations	14	14
R-squared	0.633	0.701
F-stat	9.47	12.92
Prob F	0.0041	0.00130

Standard errors in parentheses
[a.] $p < 0.01$, [b] $p < 0.1$.

which implies that $\frac{\partial \mu_2^s(\cdot)}{\partial n} > 0$ always, and that $\frac{\partial \mu_1^s(\cdot)}{\partial n} > 0$ when ϕ or n are low enough. In other words, the more different in size the two countries are, the stronger the incentives for the government of the smaller one towards fiscal laxity; in an extreme asymmetry, those incentives apply for both governments. As it turns out, if we had used this model to predict the future of the euro, we would have predicted not only the crisis, but also the identities of the countries in each side of the cunnundrum.

Table 2

PANEL FIXED-EFFECT REGRESSION ON DEFICIT FOR EURO AREA MEMBERS

	(1)	(2)
Covariates	*1986-2011* Log(Def)	*10 years of entrance* Log(Def)
Debt/GDP	0.0145b	0.0849a
	(0.00605)	(0.0199)
Pop	−0.0111	−0.360
	(0.0395)	(0.218)
Constant	0.394	5.779
	(0.833)	(4.969)
Observations	383	275
R-squared	0.045	0.14
F-stat	3.039	9.861
Prob F	0.0674	0.00210

Standard errors in parentheses ^{a} $p<0.01$, ^{b} $p<0.05$.

5. CONCLUSIONS

In the discussion leading to the creation of the euro, a number of voices were raised about the damaging incentives this would imply for the poorer countries in the zone, once they shared a currency with the richer and traditionally more fiscally prudent countries in the North of the zone. In particular, in Britain, the argument that in an eventual crisis in a Mediterranean country, the British Treasure would be expected to collaborate with the German one in funding the bailout, was voiced often and part of the reasons why the country eventually chose to opt out. Not surprisingly, Denmark also opted out, and Sweden has dragged its feet regarding euro membership, while the poorer new members are in general very keen to be part of the currency union.

In the early history of the United States, something similar happened. While the dollar as a unit of account existed before independence, the US chose to postpone minting a federal currency –and outlawing of state and private ones– for almost a century, until 1863, and the creation of the Federal Reserve Bank as a single central bank for the whole nation only happened in the early 20th century. These events, when they happened, were preceded by Acts that limited somehow the deficits that the state and local governments could run and finance without Federal authorization. Somehow, Jefferson and Adams understood what their european successors of two centuries later would not: that the moral hazard associated with fiscal independence and monetary union can be very damaging.

Quantitatively, these arguments carry some traction. The following tables show the results of some simple estimation that make the point. We ran two regressions, summarized in Table 1, of the relative deficit before and after entering the euro zone, with respect to the relative debt (in the same sense) and the population. We use two samples for the analysis: for the estimates in column one we include the whole 13 years before and after the inception of the euro until now, and for the estimates in column two we use 10 years before/after each countries entrance (in order to largely avoid including the current crisis in the sample).

Since the model is logarithmic in the dependent variable and the covariates, the coefficients reflect an elasticity. Interestingly, in both specifications the constant is positive (fiscal discipline became laxer in all countries upon acquiring the single currency), and the effect of the population is negative (fiscal discipline suffered more in smaller countries). Moreover, in the reduced-sample model all the covariates are significant at the 1% level, while in the first the debt is only significant at the 10%. The R^2 and F-statistic are presented as global robustness measures.

Instead of using cross-section data, we build up a panel with information on fiscal deficit, debt and population for each country, and display the results in Table 2. Here, only the dependent variable is logarithmic, while all the covariates are in levels, so the coefficient stands for a semi-elasticity. As before, the effect of the debt is positive and the population is negative, though the latter is not significant now. Nonetheless, the global F-statistic shows that models are robust (with a better performance when the reduced-model is estimated).

In this paper, we have illustrated a theoretical model in which the use of money emerges endogenously, which in turn is derived from a previous model in which the advantages in trade of a common currency are also explicit and endogenous, and in which an equilibrium where all monies in existence circulate everywhere and become interlinked is not only more robust than other equilibrium, but also superior. Nevertheless, these advantages disappear when one introduces the possibility of independent taxation or expenditures decisions in each nation. Potential areas of extension for this work include the endogenization of M, the application of the same ideas in a divisible-money set up –analogous to Lagos and Wright (2005), perhaps– or the generalization of the model to cases where the countries can be different in the efficiency of their local market or in their productivity. Some preliminary analysis leads us to believe that, in the latter case, asymmetries in productivity or in market technology would push in the same direction as those in size: the moral hazard on the poorer economies government would worsen. Moreover, it may be interesting to consider other forms of taxation by the local governments –and discuss which of them makes a more relevant analogy to the process of extracting seigniorage from a common currency.

In equilibrium, more seigniorage is extracted by a government interested in maximizing seigniorage than by one interested in maximizing welfare. More interestingly, two governments that coordinate their actions can extract higher seigniorage, while choosing lower seigniorage rates, by avoiding the seigniorage wars that take place when the strategic interaction among them leads to an inefficient equilibrium. This inefficiency is larger when the populations or productivities are very asymmetric. In particular, a very small or poor country would always take its seigniorage collection to the highest possible rate. The lack of a European Central Treasury, and a binding Fiscal Compact, given the existence of the European Central Bank, makes this inefficiency come to light, and the recent events in Greece, Cyprus, Portugal and, to a lesser extent, other Mediterranean nations, are a real manifestation of it.

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