# The Art of Monetary Theory: A New Monetarist Perspective*

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> This, as I see it, is really the central issue in the pure theory of money. Either we have to give an explanation of the fact that people do hold money when rates of interest are positive, or we have to evade the difficulty somehow... The great evaders ... would have put it down to "frictions," and since there was no adequate place for frictions in the rest of their economic theory, a theory of money based on frictions did not seem to them a promising field for economic analysis. This is where I disagree. I think we have to look frictions in the face, and see if they are really so refractory after all. Hicks (1935)

> Progress can be made in monetary theory and policy analysis only by modeling monetary arrangements explicitly. Kareken and Wallace (1980)

# 1 Introduction

Over the past 25 years a new approach has been developed to study monetary theory and policy, and more broadly to study liquidity. This approach sometimes goes by the name New Monetarist Economics.[1] Research in the area lies at an interface between macro and micro – it is meant to be empirically and policy relevant, but it also strives for theoretical rigor and logical consistency. While most economists want to be rigorous and consistent, we would argue that those we call New Monetarists are *more* concerned with microfoundations than alternative schools in macro. And unlike most micro theorists, they neither ignore money nor use the word as a synonym for transferable utility. While often abstract, the research has become increasingly oriented toward policy, partly because the theories have matured, and partly because recent events have put monetary matters front and center, including those related to interest rates, banking, credit conditions, financial markets and liquidity.

Papers in the area are diverse, yet share a set of principles and methods. Questions include: What is money, why do we use it, and is it essential? Which objects will (or should) play this role in equilibrium (or optimal) arrangements? How is intrinsically worthless currency valued, or more generally, how can asset prices differ from "fun-

[1] A discussion of the New Monetarist label is contained in Williamson and Wright (2010*a*). Briefly, at least some people in this camp find attractive many, but by no means all, elements of the Old Monetarist school represented by Friedman (1960,1968,1969) and his followers. The appellation also suggests an opposition to New Keynesian Economics, partly for its disregard for microfoundations, and partly for its focus on nominal rigidities as *the* critical (perhaps exclusive) distortion relevant for theoretical, empirical and policy analysis. We hope it is a constructive opposition, the way it was good for Old Keynesians to have Old Monetarists questioning their doctrine. While we do not want to make too much of nomenclature, we believe it is useful to get the language right when trying to understand and differentiate alternative ways to think about monetary economics.

damental" values? How does credit work absent commitment? How can credit and money coexist? What is the role of assets in credit arrangements? What are the roles of intermediation, and of inside and outside money? What are the effects of inflation, or monetary policy, more broadly? What is optimal policy? As should be clear, the research is about much more than pricing currency. It is about trying to understand the process of exchange in the presence of frictions, and how this process might be facilitated by institutions, including money, but also credit, intermediation, and the use of assets as payment instruments or as collateral.

A defining characteristic unifying the work discussed below is that it models the exchange process explicitly, in the sense that *agents trade with each other*, at least in some if not all situations. That is not true in GE (general equilibrium) theory, where agents only "trade" against their budget lines. In the Arrow-Debreu paradigm, agents are endowed with a vector $\bar{\mathbf{x}}$, and choose another $\mathbf{x}$ subject only to $\mathbf{px} \leq \mathbf{p\bar{x}}$, taking as given the vector $\mathbf{p}$; how they get from $\bar{\mathbf{x}}$ to $\mathbf{x}$ is not up for discussion.[2] The models surveyed here go beyond classical micro by incorporating explicit frictions that hinder interactions between agents. It is only then that one is able to analyze how institutions ameliorate these frictions. Since money is one such institution, and maybe the most elemental, it is natural to start there, but we do not stop there. The focus is much broader, and includes studying any institution whose raison d'être is the facilitation of exchange. For this one needs explicit models of the trading process.[3]

[2] Earlier work surveyed by Ostroy and Starr (1990) asked some of the right questions, but did not resolve all the issues. Also related is work trying to use Shapley and Shubik's (1969) market games as a foundation for monetary economics, e.g. Hayiashi and Matsui (1996).

[3] Others share these concerns. As regards "the old conundrum of how fiat money can survive as an institution" Hahn (1987) says: "At a common-sense level almost everyone has an answer to this, and old-fashioned textbooks used to embroider on some of the banalities at great length. But common sense is, of course, no substitute for thought and certainly not for theory. In particular, most of the models of an economy which we have, and I am thinking here of many besides those of Arrow and Debreu, have no formal account for the exchange process." Similarly, Clower (1970) says: "conventional value theory is essentially a device for logical analysis of virtual trades in a world where individual economic activities are costlessly coordinated by a central market authority. It has nothing whatever to say about delivery and payment arrangements, about the timing or frequency of market transactions, about search, bargaining, information and other trading costs, or about countless other commonplace features of real-world trading processes." On the related topic of intermediation, Rubinstein and Wolinsky (1987) say: "Despite the important role played by intermediation in most markets, it is largely ignored by the standard theoretical literature. This is because a study of intermediation requires a basic model that describes explicitly the trade frictions that give rise to the function of intermediation. But this is missing from the standard market models, where the actual process of trading is left unmodeled."

The literature we survey borrows from GE theory, but also relies heavily on search theory, which is all about agents trading with each other. It also uses game theory, naturally, since many of the issues are inherently strategic – e.g., what you use as a medium of exchange depends on what others use. At the intersection of search and game theory, the models often use bargaining, which leads to insights one might miss if exclusively devoted to a Walrasian auctioneer. Other ways to determine the terms of trade are also studied, including price taking, posting and more abstract mechanisms. Key frictions include spatial and temporal separation, limited commitment and imperfect information. These can make arrangements involving money or intermediation socially beneficial, which is *not* the case in approaches taking short-cuts, like CIA (cash-in-advance) or MUF (money-in-utility-function) models. Such reduced-form devices are presumably meant to stand in for the idea that assets help overcome difficulties in exchange, something we want to make explicit.

Moreover, in CIA models, having to use cash hurts people; in sticky-price models, all the problems arise from forcing agents to set prices in dollars and making them hard to change. If money were really such a hindrance how did it survive all these centuries? In contrast, the literature reviewed here uses models where relevant institutions arise endogenously, including monetary and credit arrangements. This gives different answers than reduced-form models, and allows one to address additional questions. How can one purport to understand financial crises or banking problems, e.g., using theories with no role for payment or settlement systems in the first place? How can one hope to assess the effects of inflation, e.g., which is a tax on the use of money, using theories that do not incorporate the frictions money is supposed overcome? To be clear, we do *not* claim the models here provide answers to all the important questions; we *do* think they provide a useful way to think about them.

Notable early work in monetary theory includes OLG (overlapping-generations) papers by Samuelson (1958), Lucas (1972) and Wallace (1980). Since Kiyotaki and Wright (1989), more monetary economists employ search theory (other early attempts to model money using search are Jones 1976, Oh 1989 and Iwai 1996). But spatial separation per se is not crucial: as shown by Kocherlakota (1998), with some credit to Ostroy (1973) and Townsend (1987*b*), money is useful when it ameliorates double-coincidence problems under imperfect commitment and information; search is just one way to formalize this. In banking, work that fits under the New Monetarist umbrella includes Berentsen et al. (2007) and Williamson (2012), some of which naturally draws on earlier research by Diamond and Dybvig (1983), Diamond (1984) and Williamson (1986,1987). Models of intermediation and finance include Rubinstein and Wolinsky (1987), Duffie et al. (2005) and Lagos and Rocheteau (2009). Models of credit include Kehoe and Levine (1993) and Kiyotaki and Moore (1997). Further delineating our approach, the idea of modeling institutions, including monetary arrangements, endogenously is related to the Lucas (1976) critique, the Wallace (1998) dictum, and a methodology espoused in Townsend (1987*a*,1988).[4]

As a preview, we start with first-generation search models of money studied by Kiyotaki and Wright (1989,1993), Aiyagari and Wallace (1991,1992) and others, to illustrate the tradeoff between asset returns and acceptability, and to show how economies where liquidity plays a role are prone to multiplicity and volatility. They can also be used to present a simplified version of Kocherlakota's (1998) results on the essentiality of money, and the analysis of inside and outside money in Cavalcanti and Wallace (1999*a*), both of which are related to Kehoe and Levine (1993). We then move to the second-generation models studied by Shi (1995), Trejos and Wright (1995) and others, with divisible goods. This allows us to further explore the efficiency of monetary exchange, and the idea that liquidity considerations can lead to multiplicity and volatility. We also discuss the relation to the intermediation models of Rubinstein and Wolinsky (1987) and Duffie et al. (2005).

We then move to divisible assets, as in the computational work of Molico (2006), or the more tractable models Shi (1997*a*) and Lagos and Wright (2005). These models are easier to integrate with mainstream macro, and thus allow us to study some standard issues in a new light (e.g., the cost of inflation). We also discuss the effects of monetary policy on labor markets, the interaction between money and other assets in facilitating exchange either as a payment instrument or as collateral, and the theme that observations that seem anomalous from the perspective of standard asset pricing emerge naturally in models with trading frictions. Then, just as some models relax the indivisible asset assumption in earlier search-based monetary theory, we show how

[4] In case readers do not know, Wallace's dictum is this: "*Money* should not be a primitive in monetary theory – in the same way that *firm* should not be a primitive in industrial organization theory or *bond* a primitive in finance theory."

Lagos and Rocheteau (2009) relax it in models of OTC (over-the-counter) markets like Duffie et al. (2005). Finally, we discuss some information-based models.

Before getting into theory, let us clarify a few terms and put them in historical perspective. First, when agents trade with each other, it is commonly understood that there may be a *double-coincidence problem* with direct barter: in a bilateral meeting between individuals $i$ and $j$, it would be a *coincidence* if $i$ liked $j$'s good, and a *double coincidence* if $j$ also liked $i$'s good.[5] Next, we make reference to a *medium of exchange*, for which it is hard to improve on Wicksell (1911): "an object which is taken in exchange, not for its own account, i.e. not to be consumed by the receiver or to be employed in technical production, but to be exchanged for something else within a longer or shorter period of time." Sometimes we similarly use *means of payment*. Relatedly, a *settlement instrument* is something that discharges a debt.

A medium of exchange that is also a consumption or production good is *commodity money*. By contrast, as Wallace (1980) puts it, *fiat money* is a medium of exchange that is intrinsically useless (neither a consumption nor a productive input) and inconvertible (not a claim redeemable in consumption or production goods). This usefully defines a pure case, although assets other than fiat currency can convey *moneyness*. Finally, one of the main challenges in monetary theory is to describe environments where an institution like money is *essential*. As introduced by Hahn (1973), essentiality means welfare is higher, or the set of incentive-feasible allocations is larger, with money than without it. That this is nontrivial is evidenced by the fact that money is plainly *not* essential in most standard theories. As Debreu (1959) says about his theory, an "important and difficult question ... not answered by the approach taken here: the integration of money in the theory of value." We hope the essay clarifies what ingredients are relevant for understanding the issues, and how economics gets more interesting and useful once they are incorporated.[6]

[5] Jevons' (1875) often gets credit, but other discussions of the idea and related issues in the history of thought include von Mises (1912), Wicksell (1911), Menger (1892) and Smith (1776). Sargent and Velde (2003) say the first to recognize how money helps double-coincidence problems was the Roman Paulus in the 2nd century, despite Schumpeter's (1954) claim for Aristotle.

[6] Related surveys or discussions include Wallace (2001,2010), Wright (2005), Shi (2006), Lagos (2008), Williamson and Wright (2010$a$,$b$) and Nosal and Rocheteau (2011). By comparison: (i) we discuss liquidity in general, with money a special case; (ii) we make more connections to finance and labor; (iii) we provide simplified versions of some difficult material not in previous surveys; (iv) we present more quantitative results. Surveys of the very different New Keynesian approach include Clarida et al. (1999) and Woodford (2003); Gertler and Kiyotaki (2010) is somewhere in between.

## 2 Commodities as Money

We begin with Kiyotaki and Wright (1989), a model that can now be described more easily than when it was first developed, due to advances in technique. The goal is to derive equilibrium trading patterns endogenously, to see if they resemble arrangements observed in actual economies, in a stylized sense, and especially to see if there emerge institutions like the use of some commodities as media of exchange or some agents as intermediaries.

Time is discrete and continuous forever. There is a $[0,1]$ continuum of infinitely-lived agents that interact according to a bilateral matching process. They have specialized tastes and technologies: there are $N$ types of agents and $N$ goods, where agents of type $j$ consume good $j$ but produce good $j+1$ modulo $N$ (i.e., type $N$ agents produce good $N+1=1$). For now, the fraction of type $j$ is $n_j = 1/N$, and we set $N=3$. Although we usually call the agents consumers, we emphasize from the outset that similar considerations are relevant for producers. So, instead of saying that consumer $j$ produces good $j+1$ but wants to consume good $j$, it is a relabeling to say that firm $j$ uses input $j$ and produces $j+1$. This generates the same motives for and difficulties with trade. However, a double-coincidence problem does nothing to rule out credit. For that we need a lack of exogenous enforcement and commitment, plus imperfect information, such as private trading histories. As discussed in Section 3, only under these conditions is there a need indirect trade using media of exchange.

Goods are indivisible and storable, but only 1 unit at a time. Let $\rho_j$ be the return on good $j$, by which we mean the flow utility agents get when they have a unit of it in inventory. One can interpret $\rho_j > 0$ as a dividend (e.g., "fruit" from a Lucas 1978 "tree"), and $\rho_j < 0$ as a storage cost. As discussed in many places (e.g., Nosal and Rocheteau 2011, Chapter 5), it is a venerable idea that the intrinsic properties of objects influence which will, or should, serve as media of exchange, and storability is the property in focus here. Type $j$ also gets utility $u > 0$ when he consumes good $j$, and then produces a new unit of good $j+1$ at cost normalized to $c = 0$. Or, relabeling agents as producers, one could say type $j$ use input $j$ to produce final output that is consumed for utility $u$. Type $j$ always accepts good $j$ in trade, and consumes it, at least if $|\rho_j|$ is not too big (as discussed below, if $|\rho_j|$ is too big agents may dispose of good $j$ or hoard it for the dividend).

Given this, we concentrate on the following aspect of strategies: Will type $j$ trade their production good $j+1$ for good $j+2$ in an attempt to facilitate future acquisition of good $j$? Or will type $j$ hold onto good $j+1$ until it can be traded directly for consumption? Let $\tau_j$ be the probability type $j$ trades good $j+1$ for good $j+2$, and let a symmetric, stationary strategy profile be a vector $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$. If $\tau_j > 0$, type $j$ uses good $j+2$ as a medium of exchange. We also need to determine the distribution of inventories. Since type $j$ consume good $j$ when they get it, they always have either good $j+1$ or $j+2$. Hence, $\mathbf{m} = (m_1, m_2, m_3)$ characterizes the distribution, where $m_j$ is the proportion of type $j$ holding their production good $j+1$. The probability per unit time that type $i$ meets type $j$ with good $j+1$ is $\alpha n_j m_j$, where $\alpha$ is the baseline arrival rate (the probability of meeting anyone) and $n_j = 1/N$. The Appendix shows how to derive the SS (steady state) condition for each type $j$,

$$m_j n_{j+1} m_{j+1} \tau_j = (1 - m_j) n_{j+2} m_{j+2}. \tag{1}$$

To describe payoffs, let $r$ be the rate of time preference and $V_{ij}$ the value function of type $i$ holding good $j$. For convenience, let the utility from dividends be realized next period. Then, for type 1,

$$\begin{aligned} rV_{12} &= \rho_2 + \alpha n_2 (1 - m_2) u + \alpha n_3 m_3 \tau_3 u + \alpha n_2 m_2 \tau_1 (V_{13} - V_{12}) \qquad (2) \\ rV_{13} &= \rho_3 + \alpha n_3 m_3 (u + V_{12} - V_{13}), \qquad (3) \end{aligned}$$

which are standard flow DP (dynamic programming) equations.[7] The BR (best response) conditions are: $\tau_1 = 1$ if $V_{13} > V_{12}$; $\tau_1 = 0$ if $V_{13} < V_{12}$; and $\tau_1 = [0, 1]$ if $V_{13} = V_{12}$. A calculation implies $V_{13} - V_{12}$ takes the same sign as

$$\Delta_1 \equiv \rho_3 - \rho_2 + \alpha \left[ n_3 m_3 (1 - \tau_3) - n_2 (1 - m_2) \right] u, \tag{4}$$

[7] For those less familiar with search, consider $V_{12}$. Given dividends are realized next period,

$$\begin{aligned} (1 + r) V_{12} &= \rho_2 + \alpha n_1 V_{12} + \alpha n_2 m_2 \left[ \tau_1 V_{13} + (1 - \tau_1) V_{12} \right] + \alpha n_2 (1 - m_2)(u + V_{12}) \\ &\quad + \alpha n_3 m_3 \left[ \tau_3 (u + V_{12}) + (1 - \tau_3) V_{12} \right] + \alpha n_3 (1 - m_3) V_{12}. \end{aligned}$$

The RHS is type 1's payoff next period from the dividend, plus the expected value of: meeting his own type with probability $n_1$, which implies no trade; meeting type 2 with their production good with probability $n_2 m_2$, which implies trade with probability $\tau_1$; meeting type 2 with good 1, which implies trade for sure; meeting type 3 with good 1, which implies trade with probability $\tau_3$; and meeting type 3 with good 2, which implies no trade. Algebra leads to (2). Note also that the same equations apply in continuous time with $\alpha$ interpreted as a Poisson arrival rate.

In (4), $\rho_3 - \rho_2$ is the *return differential* from holding good 3 rather than good 2. If returns were all that agents valued, this would be the sole factor determining $\tau_1$. But the other term captures the difference in the probability of acquiring 1's desired good when holding good 3 rather than 2, or the *liquidity differential*. Hence, whether 1 should opt for indirect exchange, by swapping good 2 for 3 whenever the opportunity arises, involves comparing the return and liquidity differentials. This reduces the BR condition for $\tau_1$ to a check on the sign of $\Delta_1$. Similarly, for any $j$,

$$\tau_j = \begin{cases} 1 & \text{if } \Delta_j > 0 \\ [0,1] & \text{if } \Delta_j = 0 \\ 0 & \text{if } \Delta_j < 0 \end{cases} \tag{5}$$

Let $\mathbf{V} = (V_{ij})$ be the vector of value functions. Then a stationary, symmetric equilibrium is a list $\langle \mathbf{V}, \mathbf{m}, \boldsymbol{\tau} \rangle$ satisfying the DP, SS and BR conditions. There are 8 candidate equilibria in pure strategies, and for each such $\tau$, one can solve for $m$, and use (5) to determine the parameters for which $\tau$ is a BR to itself (see the Appendix).

To present the results, assume $\rho_1, \rho_2 > 0 = \rho_3$, so we can display outcomes in the positive quadrant of $(\rho_1, \rho_2)$ space. Figure 1 shows different regions labeled by $\boldsymbol{\tau}$ to indicate which equilibria exist. There are two cases, Model A or B, distinguished by $\rho_1 > \rho_2$ or $\rho_2 > \rho_1$.[8] In Figure 1, Model A corresponds to the region below the $45^o$ line, where there are two possibilities: if $\rho_2 \geq \hat{\rho}_2$ the unique outcome is $\boldsymbol{\tau} = (0, 1, 0)$; and if $\rho_2 \leq \bar{\rho}_2$ the unique outcome is $\boldsymbol{\tau} = (1, 1, 0)$. To understand this, note that for type 1 good 3 is more liquid than good 2. It is more liquid because type 3 accepts good 3 but not good 2, and type 3 always has what type 1 wants ($m_3 = 1$). In contrast, type 2 agents always accept good 2, but do not always have 1's good ($m_2 = 1/2$). Hence, good 3 allows type 1 to consume more quickly. If $\rho_2 > \hat{\rho}_2$ this liquidity differential does not compensate for a lower return; if $\rho_2 < \hat{\rho}_2$ it does.[9]

In Model A, $\boldsymbol{\tau} = (0, 1, 0)$ is what Kiyotaki and Wright call the "fundamental" equilibrium, where good 1 is the universally-accepted commodity money, while type

[8] In Kiyotaki and Wright (1989), Models A and B both have $\rho_1 > \rho_2$, but in Model B type $j$ produces good $j - 1$ instead of $j + 1$. Here we keep production fixed and distinguish A or B by $\rho_1 > \rho_2$ or $\rho_2 > \rho_1$, which is equivalent but easier. To see how they differ, consider Model A with $\rho_1 > \rho_2$. Then at least myopically it looks like a bad idea for type 1 to set $\tau_1 = 1$, because trading good 2 for 3 lowers his return. Similarly, it looks like a bad idea for type 3 to set $\tau_3 = 1$, and a good idea for type 2 to set $\tau_2 = 1$. Hence, exactly one type is predisposed to use indirect trade based on fundamentals. In Model B, types 2 and 3 are both so predisposed.

[9] To see why type 1 is pivotal, note that for type 2 trading good 3 for good 1 enhances both liquidity and return, as does holding onto good 1 for type 3. Hence, only type 1 has a tradeoff.

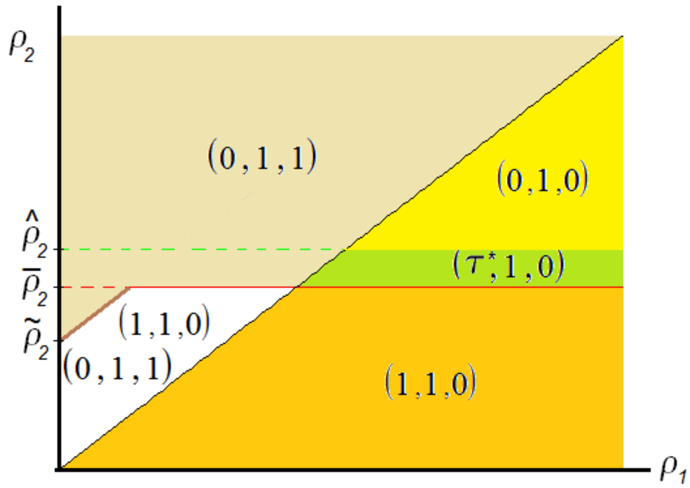


Figure 1: Equilibria with Commodities as Money

2 agents act as middlemen by acquiring good 1 from producers and delivering it to consumers. While this may seem the most natural outcome, when $\rho_2 < \bar{\rho}_2$ we instead get $\boldsymbol{\tau} = (1, 1, 0)$, called a "speculative" equilibrium, where type 1 trades the high-return good 2 for the low-return good 3 with the expectation of better future prospects. In this equilibrium good 1 and good 3 are both used as money. Theory delivers cutoffs that make type 1 willing or unwilling to sacrifice return for liquidity. Notice, however, that there is a gap between the cutoffs: for $\hat{\rho}_2 > \rho_2 > \bar{\rho}_2$ there is no stationary, symmetric equilibrium in pure-strategies; as shown, there is one in mixed-strategies, where type 1 accepts good 3 with probability $\tau^* \in (0, 1)$.[10]

Things are different in Model B, above the $45^o$ line. Given the market does not shut down, there is always an equilibrium with $\boldsymbol{\tau} = (0, 1, 1)$. This is the "fundamental" equilibrium for Model B, where type 1 hang on to good 2, which now has the highest return, while types 2 and 3 opt for indirect exchange, with goods 1 and 2 serving as money. For some parameters, as shown, there coexists an equilibrium with $\boldsymbol{\tau} = (1, 1, 0)$, the "speculative" equilibrium for this specification, where goods 3 and 1 are

[10] See Kehoe et al. (1993). They also show there can be multiple steady-state equilibria in mixed strategies, but the set of such equilibria is generically finite. And they construct equilibria where $\tau_1$ cycles, with liquidity fluctuating over time as a self-fulfilling prophecy. Trachter and Oberfield (2012) provide conditions under which the set of dynamic equilibria shrinks as the length of the discrete-time period gets small. More generally, however, we argue below that economies where liquidity plays a role are often inherently prone to multiplicity and volatility.

commodity monies, while good 2 is not universally accepted even though it now has the best return. The coexistence of equilibria with different transactions patterns and liquidity properties shows that these are not necessarily pinned down by fundamentals.

While his sums up the baseline model, there are many extensions and applications. Aiyagari and Wallace (1991) allow $N$ types and $N$ goods, prove existence of equilibria with trade, and show there always exists one equilibrium where the best good is universally accepted, although that is not true in all equilibria.[11] Another extension, considered in Kiyotaki and Wright (1989), Aiyagari and Wallace (1992) and elsewhere, is to add fiat currency. We postpone discussion of this, but mention for now that it provides one way to see that equilibria are not generally efficient: for some parameters, equilibria with valued fiat money Pareto dominate other equilibria. In terms of comparing different commodity-money equilibria when they coexist, it may *seem* better to use the highest-return object as a medium of exchange. But at least some agents can prefer to have other objects so used – especially those who produce the other objects, reminiscent of the bimetalism debates (see the entertaining discussion in Friedman 1992).

To study how we might get to equilibrium, several papers use evolutionary dynamics.[12] In Wright (1995), in particular, there is a general population, $\mathbf{n} = (n_1, n_2, n_3)$, which is interesting for its own sake (for some $\mathbf{n}$ a new equilibrium emerges). Then agents are allowed to choose their type, interpretable as choosing preferences or technologies, or as an evolutionary process where types with higher payoffs increase in numbers due to reproduction or imitation. In Model A, with $\mathbf{n}_t$ evolving according to standard Darwinian dynamics, for any initial $\mathbf{n}_0$, and any initial equilibrium if $\mathbf{n}_0$ admits multiplicity, $\mathbf{n}_t \to \mathbf{n}_\infty$ where at $\mathbf{n}_\infty$ the unique equilibrium is "speculative." While one may have thought "fundamental" play would win out, the result can be

[11] Proving the existence of equilibria with trade is nontrivial because standard fixed-point theorems do not guarantee a particular exchange pattern – one of Hahn's (1965) problems. Hence, the arguments are necessarily more involved. See also Zhu (2003,2005).

[12] These include Matsuyama et al. (1993), Wright (1995), Luo (1999) and Sethi (1999). A related literature, including Marimon et al. (1990) and Başçı (1999), asks if artificially-intelligent agents can learn to play equilibrium in the model. There are also studies with real people in the lab. In terms of these experiments, Brown (1996) and Duffy and Ochs (1999) find that subjects have little problem getting to the "fundamental" equilibrium, but can be reluctant to adopt "speculative" strategies. However, Duffy (2001) shows they can learn to do so. In the version with fiat money, Duffy and Ochs (2002) find that monetary equilibria tend to get selected, and that generally it is not hard to get objects valued for liquidity.

reconciled with intuition. In equilibrium with $\boldsymbol{\tau} = (0, 1, 0)$, type 3 enjoy the highest payoff, since they produce a good with the best return and highest liquidity. Ergo, $n_3$ increases. And, when type 1 interact with type 3 more often, they are more inclined to "speculate" by sacrificing return for liquidity.

Since some people find random matching unpalatable in monetary economic (e.g., Howitt 2005), the model has be redone with directed search. Corbae et al. (2003) use a solution concept generalizing Gale and Shapley (1962).[13] For Model A with $n_i = 1/3$, they find the "fundamental" outcome $\tau = (0, 1, 0)$ is the unique equilibrium in a certain class. On the equilibrium path, starting at $\mathbf{m} = (1, 1, 1)$, type 2 trade with 3 while 1 sit out. Next period, at $\mathbf{m} = (1, 0, 1)$, type 2 trade with 1 while 3 sit out. Then we are back at $\mathbf{m} = (1, 1, 1)$. Good 1 is always the unique money, different than random matching. Heuristically, with random matching, in "speculative" equilibrium type 1 is concerned with the chance of meeting type 3 with good 1. But with directed matching, chance is not a factor: the endogenous transaction pattern is deterministic, with type 2 catering to 1's needs by delivering goods every second period. So, some randomness is needed to make operative 1's "precautionary" demand for liquidity.[14]

An extension by Renero (1998,1999) further studies mixed strategies. He shows that in any equilibrium where all agents randomize, goods with lower $\rho$ must have higher acceptability, and this can be socially desirable. If this is surprising, note that to make agents indifferent between trading lower- for higher-return objects the former must be more liquid (it is also related to Gresham's Law). Other applications include Camera (2001), who studies intermediation in more detail, and Cuadras-Morató (1994) and Li (1995), who consider recognizability (information frictions) as an alternative to storability. More can be done, but we now move to other models. We spent time on this one because it provides an early formalization of the tradeoff between return and liquidity, and because there is no comparably accessible presentation.

[13] At each $t$, population partitions into subsets containing at most two agents (there is still bilateral trade) such that there are no profitable deviations in trades or trading partners for any individual or pair. While agents can choose a *type* to meet, it is often assumed in this approach that two *individuals* still meet with probability 0.

[14] As discussed below, there is often a reinterpretation of random matching in terms of preference/technology shocks (as Hogan and Luther 2014 discuss explicitly in this model). Also, the results for endogenous matching have only been established for a certain class of equilibria, for Model A, and for $n_i = 1/3$. It is not known what happens in Model B, with general $\mathbf{n}$, or with endogenous $\mathbf{n}$.

# 3 Assets as Money

Adding other assets allows us to more easily illustrate additional results. These include: (i) assets can facilitate intertemporal exchange; (ii) this may be true for fiat currency, an asset with a 0 return, or even assets with negative returns; (iii) for money to be essential, necessary conditions include limited commitment and imperfect information; (iv) the value of fiat money is both tenuous and robust; and (v) whether assets circulate as media of exchange may not be pinned down by primitives. The basic setup follows Kiyotaki and Wright (1991,1993), simplified it in a few ways.[15]

Goods are now nonstorable: they are produced on the spot, for immediate consumption, at cost $c > 0$. Hence, they cannot be retraded (one can think of them as services). Agents still specialize, but now, when $i$ and $j$ meet, the probability of a double coincidence is $\delta$, the probability of a single coincidence where $i$ likes $j$'s output is $\sigma$, and the probability of a single coincidence where $j$ likes $i$'s output is $\sigma$. SO, e.g., the double-coincidence meeting rate is $\alpha\delta$, with $\alpha$ and $\delta$ capturing search and matching, respectively. Note the equations below hold in discrete time, or in continuous time with $\alpha$ interpreted as a Poisson parameter. Any good $i$ likes gives him utility $u > c$; all others give him 0. There is a storable asset that no one consumes, but it yields a dividend, i.e. flow utility $\rho$. If $\rho = 0$ it is fiat currency. We provisionally assume agents neither dispose of nor hoard assets, and check it below.

In general, $A \in [0, 1]$ is the fixed asset supply, and we continue to assume assets are indivisible and agents can hold at most 1 unit, $a \in \{0, 1\}$. To begin, however, let $A = 0$, so that barter is the only option. The flow barter payoff is $rV^B = \alpha\delta\,(u - c)$. If $\delta > 0$ this beats autarky, $V^B > V^A = 0$, but as long as $\sigma > 0$ it does not do as well as we might like: in some meetings, $i$ wants to trade but $j$ will not oblige, which is bad for everyone in the long run. Suppose we try to institute a *credit* system, where agent $i$ produces for $j$ whenever $j$ likes his output. This is called credit because agents produce while receiving nothing by way of quid pro quo, except an understanding – call it a promise – that someone will do the same for them in the future. The flow

[15] In the original model, as in Diamond (1982), agents go to one "island" to produce and another to trade; here they produce on the spot. Also, in early versions, agents consume all goods, but like some goods more than others, so they have to choose which to accept; here the choice is trivial. Also, in early formulations agents with assets cannot produce, and so must use money even in double-coincidence meetings; following Siandra (1990), here they can barter whenever they like.

payoff to such an arrangement is

$$rV^C = \alpha\delta\,(u - c) + \alpha\sigma u - \alpha\sigma c = \alpha\,(\delta + \sigma)\,(u - c)\,.$$

If $\sigma > 0$ then $V^C > V^B$. So, if agents can commit, they can promise to abide by the credit regime, and this achieves efficiency conditional on the matching process.

If they cannot commit, promises must be credible (Kehoe and Levine 1993). This implies an incentive-compatibility condition, IC for short. The potentially binding condition applies to production in single-coincidence meetings. Whether this ensues depends on the consequences of deviating. If $V^D$ is the deviation payoff, the IC is

$$-c + V^C \geq \mu V^D + (1 - \mu)\,V^C, \tag{6}$$

where $\mu$ is the probability deviators are caught and punished: $\mu < 1$ captures imperfect monitoring, or record keeping, so that deviations are only probabilistically detected and communicated to the population at large. Suppose $V^D = V^B$, so that punishment entails a loss of future credit (one can also consider autarky). Then (6) holds iff

$$c \leq \frac{\mu\alpha\sigma u}{r + \mu\alpha\sigma} \equiv \hat{c}_C. \tag{7}$$

Naturally, for credit to be viable the monitoring/communication technology must be sufficiently good (i.e., $\mu$ cannot be too small). Suppose credit is not viable, and consider using money. Let $V_a$ be the value function for agents with $a \in \{0, 1\}$, and call those with $a = 1$ buyers and those with $a = 0$ sellers. Then

$$rV_0 = \alpha\delta\,(u - c) + \alpha\sigma A\tau_0\tau_1\,(V_1 - V_0 - c) \tag{8}$$

$$rV_1 = \alpha\delta\,(u - c) + \alpha\sigma\,(1 - A)\,\tau_0\tau_1\,(u + V_0 - V_1) + \rho, \tag{9}$$

where $\tau_0$ is the probability a seller is willing to produce for an asset while $\tau_1$ is the probability a buyer is willing give up an asset, and we include $\rho$ so the equations also apply to real assets. If $\Delta = V_1 - V_0$, the BR conditions are:

$$\tau_0 = \begin{cases} 1 & \text{if } \Delta > c \\ [0,1] & \text{if } \Delta = c \\ 0 & \text{if } \Delta < c \end{cases} \text{ and } \tau_1 = \begin{cases} 1 & \text{if } u > \Delta \\ [0,1] & \text{if } u = \Delta \\ 0 & \text{if } u < \Delta \end{cases} \tag{10}$$

Letting $\mathbf{V} = (V_0, V_1)$ and $\boldsymbol{\tau} = (\tau_0, \tau_1)$, equilibrium is a list $\langle \mathbf{V}, \boldsymbol{\tau} \rangle$ satisfying (8)-(10). Taking for granted $\tau_1 = 1$, which as shown below is valid if $|\rho|$ is not too big, for now we only have to check the condition for $\tau_0 = 1$. This reduces to

$$c \leq \frac{\alpha\sigma(1-A)u}{r+\alpha\sigma(1-A)} \equiv \hat{c}_M. \tag{11}$$

If $c \leq \hat{c}_M$ monetary equilibrium exists, and it yields higher welfare than barter but lower than credit. To see this, let $W \equiv AV_1 + (1-A)V_0$, and check $V^C > W$. Also, notice $\hat{c}_C > \hat{c}_M$ iff $\mu > 1-A$. This illustrates a result in Kocherlakota (1998): if the monitoring/record-keeping technology – what he calls *memory* – is perfect, in the sense of $\mu = 1$, then money is not essential. This is because $\mu = 1$ implies $\hat{c}_C > \hat{c}_M$, so if monetary exchange is viable then credit is, too, and $V^C > W$. Credit is better because with money: (i) potential sellers may have $a = 1$; and (ii) potential buyers may have $a = 0$. With random matching, at least (ii) is robust.[16]

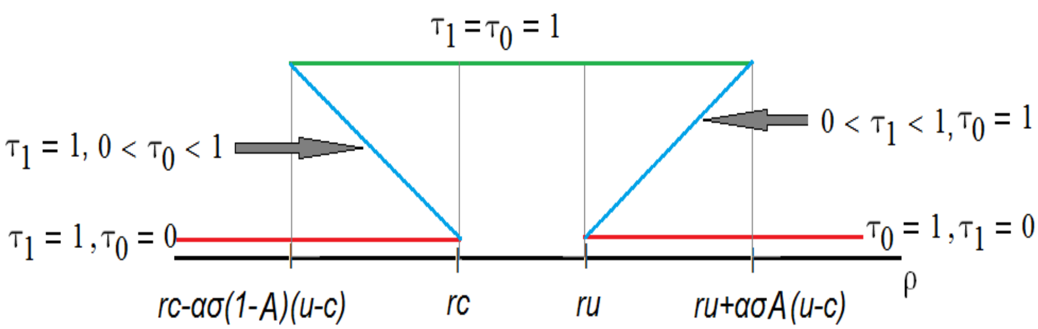


Figure 2: Equilibria with Assets as Money

To summarize, fiat money is inessential when $\mu = 1$ but may be essential when $\mu < 1$ – e.g., $\mu < 1-A$ implies $\hat{c}_C < \hat{c}_M$, so for some $c$ money works while credit does not. The necessary ingredients for essentiality are: (i) not all gains from trade are exhausted by barter; (ii) there is lack of commitment; and (iii) monitoring is imperfect.

While it can be a useful institution, fiat money is in a sense *tenuous*: there is always an equilibrium where it is not valued (in Wright 1994, there are also sunspot

[16] Actually, with directed search, money may be as good as credit, but it cannot beat credit (Corbae et al. 2003), unless there is some value to privacy that makes record keeping undesirable (Kahn et al. 2005; Kahn and Roberds 2008. Also, notice from (11) and (7), $\mu$ does not appear in the latter, since monetary exchange requires no record keeping. One might argue some record keeping is always inherent in monetary exchange, because information is conveyed by the fact that one has currency – it at least suggests having produced for someone in the past. But one could counter that sellers do not care if buyers got assets by fair means or foul (e.g., theft); they only care about others accepting it in the future.

equilibria where $\tau_0$ fluctuates). Yet money is in another sense robust: equilibria with $\tau_0 = 1$ exist even if $\rho < 0$, as long as $|\rho|$ is not extreme. To expand on this, and to check the provisionally maintained BR condition for $\tau_1 = 1$, Figure 2 shows set of equilibrium $\boldsymbol{\tau} = (\tau_0, \tau_1)$ for any $\rho$. Equilibrium $\boldsymbol{\tau} = (1, 1)$ exists iff $|\rho|$ is not too big; $\boldsymbol{\tau} = (0, 1)$ exists if $\rho < rc$, in which case buyers are willing to trade assets but sellers do not oblige; and $\tau = (1, 0)$ exists if $\rho > ru$, in which case sellers would trade but buyers won't. Thus, whether the asset circulates as a means of payment is not always pinned down by primitives: for some $\rho$ there coexist equilibria where it does, and where it does not, and mixed-strategy equilibria.[17]

For this discussion it is important that the population is large. With a finite set of agents, Araujo (2004) shows that even if deviations cannot be communicated to the population at large, one can sometimes use social norms and contagion strategies to enforce credit: if someone deviates by failing to produce for you, then you stop producing for others, they stop producing for others, and eventually this gets back to the first deviator. This can dissuade the original deviation if the population is small, but not if it is large, consistent with the stylized fact that institutions like money are used in large or complicated societies, but not always in small or primitive ones.

Even with a large population, we need imperfect monitoring, and $\mu < 1$ is one just way of modeling this.[18] In Cavalcanti and Wallace (1999*a*,*b*), a fraction $n$ of the population, sometimes called bankers in applications, monitored in all meetings; the rest, called nonbankers or anonymous agents, are never monitored. Assume that agents can now issue *notes*, say, pieces of paper with names on them, with coupon $\rho = 0$, but potentially with value as media of exchange. Notes issued by anonymous agents are never accepted by other anonymous agents in payment – why produce to get a note when you can print your own for free? But notes issued by monitored agents may be accepted in payment. Cavalcanti and Wallace use the setup to compare

[17] For an arbitrary $\tilde{\boldsymbol{\tau}}$ used by others, payoffs are $V_0(\tilde{\boldsymbol{\tau}})$ and $V_1(\tilde{\boldsymbol{\tau}})$, and (10) gives the BR correspondence, say $\boldsymbol{\tau} = \Upsilon(\tilde{\boldsymbol{\tau}})$. Equilibrium is a fixed point. Notice how $\Upsilon(\cdot)$ captures the idea that your trading strategy depends on those of others. In Figure 2, when a seller accepts $a$ he is concerned about getting stuck with it, and when a buyer gives $a$ up he is concerned about getting stuck without it, both of which depend on others' strategies. This seems relevant for many assets, e.g., real estate.

[18] The particular random monitoring formulation we use here follows Gu et al. (2013*a*,*b*). Cavalcanti and Wallace (1999*a*,*b*) have some agents monitored and not others. Sanches and Williamson (2010) have some meetings monitored and not others. Kocherlakota and Wallace (1998) and Mills (2008) assume information about deviations is detected with a lag. Amendola and Ferraris (2013) assume information about deviations is sometimes lost (forgotten).

an outside money regime, with fiat currency and no notes, to one with inside money with only notes, a classic issue in the design and regulation of banking systems.

The outside money regime is similar to the baseline model, except we can now exploit the fact that some agents are monitored. Let $W$ be average utility, and for the sake of illustration, assume monitored agents never hold money – which may be bad for them, but for a given $A$ one can show that it does not affect $W$. Let us try to implement an outcome where monitored agents produce for anyone that likes their output. To ease the presentation, set $\delta = 0$ to rule out barter. Then a monitored agent's flow payoff is $\alpha\sigma(nu - c)$, since he produces for anyone but can only consume when he meets another monitored agent, given that anonymous agents do not produce without quid pro quo. In some applications, monitored agents can always become anonymous, but suppose here that they can be punished by autarky. Then their IC is $c \leq \alpha\sigma nu/(r + \alpha\sigma)$.

For anonymous agents,

$$
\begin{aligned}
rV_0 &= \alpha\sigma nu + \alpha\sigma(1-n)(1-A)(u + V_1 - V_0) && (12)\\
rV_1 &= \alpha\sigma nu + \alpha\sigma(1-n)A(-c + V_0 - V_1), && (13)
\end{aligned}
$$

which modifies (8)-(9) by recognizing that they can consume but do not have to produce when they meet monitored agent, and $A$ now denotes the asset supply per nonmonitored agent. Generalizing (11), a monitored agent's IC is

$$
c \leq \frac{\alpha\sigma(1-n)(1-A)u}{r + \alpha\sigma(1-n)(1-A)}. \tag{14}
$$

Welfare (average utility across monitored and nonmonitored agents) is maximized at $A^* = 1/2$ because, as in the baseline model, this maximizes trade volume.

In the other regime, with no outside money, monitored agents can issue notes. This allows them to consume when they meet nonmonitored agents, which is good for $W$. Also, one can assume that with some probability bankers require nonmonitored agents with notes to turn them over to get goods, which can be used to adjust the supply of notes in circulation. Again $A = 1/2$ is optimal. It is easy to check $W$ is higher with inside money, because it lets monitored agents trade more often, by issuing notes as needed. While this may not be too surprising, the virtue of the method in general is that it allows us to concretely discuss the relative merits of different arrangements.

See Mills (2008), Wallace (2010,2013,2014), Wallace and Zhu (2007) and Deviatov and Wallace (2014) for applications.

First-generation models are crude, mainly because everything is indivisible.[19] Yet they nicely illustrate how money can be beneficial and clarify the key frictions. In other applications, Kiyotaki and Wright (1993), Camera et al. (2003) and Shi (1997*b*) endogenize specialization to show how money enhances productive efficiency. Burdett et al. (1995) analyze who searches, buyers or sellers, while Li (1994,1995) considers taxing money to correct search externalities. Ritter (1995), Green and Weber (1996), Lotz and Rocheteau (2002) and Lotz (2004) consider introducing new currencies, while Matsuyama et al. (1993) and Zhou (1997) discuss international currencies. Corbae and Ritter (2004) analyze credit. Cavalcanti et al. (1999,2005), Cavalcanti (2004), He et al. (2005) and Lester (2009) discuss banking. Araujo and Shevchenko (2006) and Araujo and Camargo (2006,2008) study learning about $A$, which is elegant because with bilateral matching information diffuses through the population slowly. So, while crude, the models are useful and have many implications that ring true.

# 4 The Terms of Trade

Second-generation monetary search theory introduced by Shi (1995) and Trejos and Wright (1995) incorporates divisible goods and let agents negotiate over the terms of trade. We now develop this model and use it to further illustrate how liquidity can give rise to multiplicity and dynamics, and to say more about efficiency.

In continuous time, which makes dynamics easier, the DP equations are

$$rV_0 = \alpha\delta\left[u(Q) - c(Q)\right] + \alpha\sigma A\tau_0\tau_1\left[V_1 - V_0 - c(q)\right] + \dot{V}_0 \quad (15)$$

$$rV_1 = \alpha\delta\left[u(Q) - c(Q)\right] + \alpha\sigma(1-A)\tau_0\tau_1\left[u(q) + V_0 - V_1\right] + \rho + \dot{V}_1, \quad (16)$$

where $\dot{V}_0$ and $\dot{V}_1$ are derivatives wrt $t$. These are like (8)-(9), except now $u = u(q)$ is the utility from $q$ units of one's consumption good and $c = c(q)$ is the disutility

[19] This does drive some results. Shevchenko and Wright (2004) argue that any equilibrium with partial acceptability, $\tau_a \in (0,1)$, is an artifact of nothing being divisibile. However, they go on to show how adding heterogeneity yields a similar multiplicity and robust partial acceptability. Note also that indivisibilities introduce a complication: as in many nonconvex environments, agents may want to trade using *lotteries*, producing in exchange for a *probability* of getting $a$ that one can interpret as a price (Berentsen et al. 2002; Berentsen and Rocheteau 2002; Lotz et al. 2007). This is actually not too hard to handle, and generates some interesting results, but instead of getting into lotteries we soon move to divisible goods.

of production. We impose the standard conditions: $u(0) = c(0) = 0$, $u'(q) > 0$, $c'(q) > 0$, $u''(q) < 0$ and $c''(q) \geq 0$ $\forall q > 0$. Also, assume $\exists \bar{q} > 0$ with $u(\bar{q}) = c(\bar{q})$. There are two quantities to be determined in (15)-(16), $q$ in a monetary trade, and $Q$ in barter. However, since $q$ can be determined independently of $Q$, we focus on the former; one can also simply set $\delta = 0$.

Before discussing equilibrium, let's ask what's feasible. In Section 3, with indivisible goods, credit is viable iff $c \leq \hat{c}_C$ and money iff $c \leq \hat{c}_M$, where $\hat{c}_C = \mu\alpha\sigma u/(r + \mu\alpha\sigma)$ and $\hat{c}_M$ is the same except $1 - A$ replaces $\mu$. The analog here is that we can support credit where agents produce $q$ for anyone that likes their output $\forall q \leq \hat{q}_C$ and we can support exchange where agents produce $q$ for fiat money $\forall q \leq \hat{q}_M$, where $c(\hat{q}_C) = \mu\alpha\sigma u(\hat{q}_C)/(r + \mu\alpha\sigma)$ and $\hat{q}_M$ *is* the same except $1 - A$ replaces $\mu$. Thus, IC now impinges on the intensive margin (how much agents trade). The relevant version of Kocherlakota (1998) is this: $\mu = 1$ implies $\hat{q}_C > \hat{q}_M$. So money is not essential when $\mu = 1$, but if $\mu < 1 - A$ then money is essential because we can support some values of $q$ with money that we cannot support without it.

For equilibrium, there are many alternatives for determining the terms of trade. Consider first Kalai's (1977) bargaining solution, which says that when a buyer gives an asset to a seller for $q$, the one who entered the meeting with $a$ gets a share $\theta_a$ of the total surplus. Since the surpluses are $S_1(q) = u(q) - \Delta$ and $S_0(q) = \Delta - c(q)$, the Kalai solution is $S_1(q) = \theta_1[u(q) - c(q)]$, or

$$\Delta = v(q) \equiv \theta_1 c(q) + \theta_0 u(q), \tag{17}$$

given the IC's $S_1(q) \geq 0$ and $S_0(q) \geq 0$ hold, as they must in equilibrium.[20] Given this, set $\tau_0 = \tau_1 = 1$ and subtract (15)-(16) to get $\Delta$ as a function of $\dot{\Delta}$, and then use (17) to arrive at a simple differential equation $\dot{q} = e(q)$.[21] Letting $\mathbf{V} = (V_0, V_1)$, equilibrium is defined by bounded paths for $\langle \mathbf{V}, q \rangle$ satisfying these conditions, with $q \in [0, \bar{q}]$, since that is necessary and sufficient for the IC conditions. Characterizing equilibria now involves analyzing an elementary dynamical system.

[20] One can interpret $\Delta = v(q)$ as a BR condition to highlight the connection to models presented above. Solve the DP equations for $\Delta(\tilde{q})$ for an arbitrary $\tilde{q}$, and then, taking $\tilde{q}$ as given, use the bargaining solution $v(q) = \Delta(\tilde{q})$ to get $q = \Upsilon(\tilde{q})$ in a meeting. Equilibrium is a fixed point. Now, one usually thinks of best responses by *individuals*, while here it is by *pairs*, but that seems a technicality relative to the conceptual merit of connecting $q = \Upsilon(q)$ to $\boldsymbol{\tau} = \Upsilon(\boldsymbol{\tau})$ in Section 3.

[21] For the record, $v'(q)\,e(q) = [\alpha\sigma(\theta_1 - A) + r\theta_1]\,c(q) - [\alpha\sigma(\theta_1 - A) - r(1 - \theta_1)]\,u(q) - \rho$.

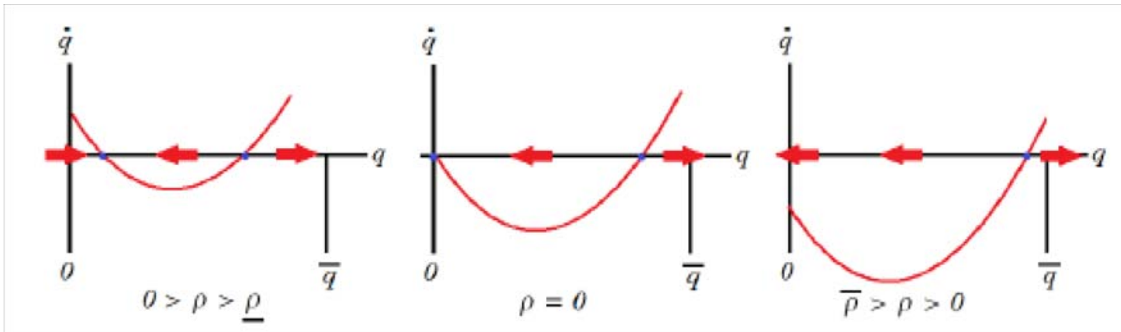


Figure 3: Assets as Money with Divisible Goods and High $\theta_1$

Trejos and Wright (2013) characterize the outcomes, depending on $\theta_1$. Figure 3 shows the case of a relatively high $\theta_1$, with subcases depending on $\rho$. Starting with $\rho = 0$ (middle panel), there are two steady states, $q = 0$ and a unique $q^e > 0$ solving $e(q^e) = 0$. There are also dynamic equilibria starting from any $q_0 < q^e$, where $q \to 0$, due to self-fulfilling inflationary expectations. For an asset with a moderate dividend (right panel), $e(q)$ shifts down, leaving a unique steady state $q^e \in (0, \bar{q})$ and a unique equilibrium, since any other solution to $\dot{q} = e(q)$ exits $[0, \bar{q}]$. If we increase the dividend further to $\rho > \overline{\rho}$ (not shown), $e(q)$ shifts further, the steady state with trade vanishes, and assets get hoarded. For a moderate storage cost $\rho \in (\underline{\rho}, 0)$ (left panel), there are two steady states with trade, $q^e_H \in (0, \bar{q})$ and $q^e_L \in (0, q^e_H)$, plus equilibria where $q \to q^e_L$ due to self-fulfilling expectations. For $\rho < \underline{\rho}$ (not shown), there is no equilibrium with trade and agents dispose of $a$.

Multiple steady states and dynamics arise because the value of a liquid asset, real or fiat, is partly self-fulfilling: if you think others give low $q_L$ for an asset then you only give a little to get it; if you think they give high $q_H$ for it then you give more. Shi (1995), Ennis (2001) and Trejos and Wright (2013) also construct sunspot equilibria where $q$ fluctuates randomly over time, while Coles and Wright (1998) construct continuous-time cycles where $q$ and $\Delta$ revolve around steady state. These results formalize the notion of excess volatility in asset values, and are different from results in ostensibly similar models without liquidity considerations (Diamond and Fudenberg 1989; Boldrin et al. 1993; Mortensen 1999), which require increasing returns in matching or production. Here dynamics are due to the self-referential nature of liquidity. Note that to get an asset sellers would incur a cost $c(q)$ above the “fundamental” value, $\rho/r$. This is a *bubble* in a standard sense: "if the reason that the price is high today is only because investors believe that the selling price is high tomorrow – when 'fundamental' factors do not seem to justify such a price – then a bubble exists" (Stiglitz 1990). One might argue liquidity services are "fundamental," too; rather than debate semantics, we emphasize the economics: liquidity considerations lead to deterministic and stochastic fluctuations with preferences and technologies constant.

In terms of efficiency, clearly $q = q^*$ *is* desirable, where $u'(q^*) = c'(q^*)$. Related to Mortensen (1982) and Hosios (1990), with fiat money and a fixed $A$, one can construct $\theta_1^*$ such that $q = q^*$ in equilibrium iff $\theta_1 = \theta_1^*$, where $\theta_1^* \leq 1$ iff $r$ is not too big. Hence, if agents are patient $\theta_1^* \leq 1$ achieves the first best; otherwise $\theta_1 = 1$ achieves the second best. One can also take $\theta$ as given and maximize $W$ wrt $A$. If $q$ is fixed, as in Section 3, the solution is $A^* = 1/2$. Now, with $q$ endogenous, if $q < q^*$ then $A^* < 1/2$ because the fall in trade volume (the extensive margin) has only a second-order cost, by the envelope theorem, while the increase in output per trade (the intensive margin) has a first-order benefit. This captures, albeit in a stylized way, the notion that monetary policy ought to balance liquidity provision and control of the price level. And although of course this depends on the upper bound for $a$, it illustrates the robust idea that the distribution of money matters.

For a diagrammatic depiction of welfare, let $S_1 = u(q) - \Delta$ and $S_0 = \Delta - c(q)$ denote the buyer's and seller's surplus. Any trade must satisfy the IC's, $S_1 \geq 0$ and $S_0 \geq 0$. The relationship between $S_1$ and $S_0$ as $q$ changes, the frontier of the bargaining set, is $S_0 = -c\left[u^{-1}(S_1 + \Delta)\right] + \Delta$ in Figure 4. Kalai's solution selects the point on the frontier intersecting the ray $S_0 = (\theta_0/\theta_1)S_1$. As $\theta_1$ increases, this ray rotates and changes $S_1$, $S_0$ and $q$. Let $S^* = u(q^*) - c(q^*)$. In the left panel of Figure 4, assuming $\Delta \geq c(q^*)$, $q^*$ can be achieved for some $\theta_1^*$ at the tangency between the frontier and the $45^o$ line. If $\theta_1 < \theta_1^*$ output is too low; if $\theta_1 > \theta_1^*$ it is too high. In in the right panel, assuming $\Delta < c(q^*)$, output is inefficiently low for all $\theta_1$, and the second best is achieved at $\theta_1 = 1$.

Many results do not rely on a particular bargaining mechanism, and various alternatives are used in the literature.[22] To nest these, let $v(q)$ be a generic mechanism

[22]Shi (1995) and Trejos and Wright (1995) use symmetric Nash bargaining while Rupert et al. (2001) use generalized Nash. They consider threat points given by continuation values and by 0, both of which can be derived from strategic bargaining in a stationary setting, as in Binmore et

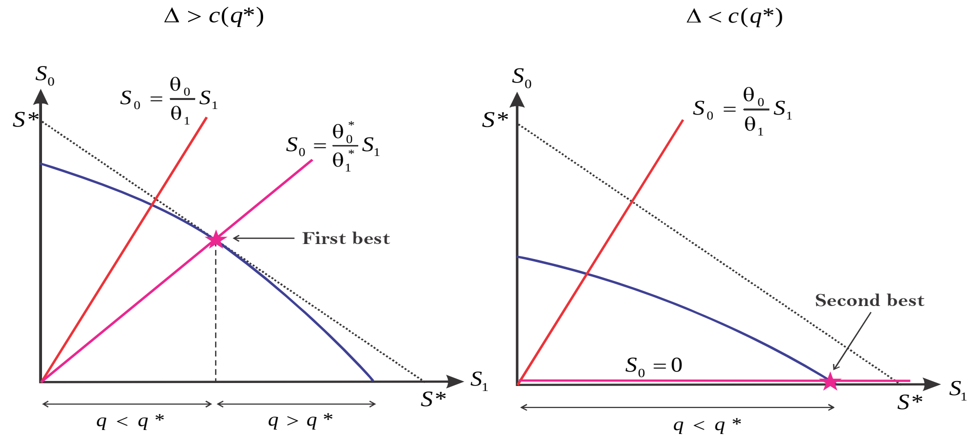


Figure 4: Proportional Bargaining and Monetary Exchange

describing how much value a buyer must transfer to a seller to get $q$. The terms of trade solve $\Delta = v(q)$. Consider e.g. generalized Nash bargaining with threat points given by continuation values: $\max_q [u(q) - \Delta]^{\theta_1} [\Delta - c(q)]^{\theta_0}$. The FOC can be written $\Delta = v(q)$ with

$$v(q) = \frac{\theta_1 u'(q) c(q) + \theta_0 c'(q) u(q)}{\theta_1 u'(q) + \theta_0 c'(q)}. \tag{18}$$

Note that (18) and (17) are the same at $q = q^*$; otherwise, given $u'' < 0$ or $c'' > 0$, they are different except in special cases like $\theta_a = 1$.

As in Trejos and Wright (1995), using Nash bargaining, we can detail how efficiency depends on various forces. First there is bargaining power; let's neutralize that by setting $\theta_1 = 1/2$. Second there is market power, coming from market tightness, as reflected in the threat points; let's neutralize that by setting $A = 1/2$. Then one can show $q < q^*$, and $q \to q^*$ as $r \to 0$. The intuition is compelling: In frictionless economies, agents work to acquire purchasing power they can turn into immediate consumption, and hence work until $u'(q^*) = c'(q^*)$. In contrast, in this model they work for assets that provide consumption only in the future, when they meet the right person. Therefore, for $r > 0$ they settle for less than $q^*$. This comes up with divisible assets, too, but it is a point worth making even with $a \in \{0, 1\}$.[23]

al. (1986). Coles and Wright (1998) use the strategic solution for nonstationary equilibria. Trejos and Wright (2013) use Kalai bargaining, which also has a strategic foundation (Dutta 2012). Others use mechanism design (Wallace and Zhu 2007; Zhu and Wallace 2007), price posting (Curtis and Wright 2004; Julien et al. 2008; Burdett et al. 2014) and auctions (Julien et al. 2008).

[23] Shi (1996), Aiyagari et al. (1996), Wallace and Zhu (2007) and Zhu and Wallace (2007) study

# 5 Intermediation in Goods Markets

Intermediaries, or middlemen, like money are institutions that facilitate trade. There are models designed to study this, and we present a well-known example due to Rubinstein and Wolinsky (1987). One reason for this is that, once we get past notation and interpretation, their model operates much like the one in Section 4, and a service a useful a survey can provide is to make connections between different applications.

There are three types, $P$, $M$ and $C$, for producers, middlemen and consumers.[24] The measure of type $i$ is $n_i$. There is a divisible but nonstorable good $q$ anyone can produce at cost $c(q) = q$ and consume for utility $u(q) = q$. Unlike Section 4, there are no gains from trade in $q$, but it can be used as a payment instrument. There are gains from trade in a different good that is indivisible and storable: type $C$ consumes it for utility $u > 0$, while type $P$ produces it at cost normalized to 0. While $M$ does not produce or consume this good, he may acquire it from $P$ to trade with $C$. It is not possible to store more than one unit, $a \in \{0, 1\}$, as above, but now the good is not in fixed total supply since type $P$ agents have an endogenous decision to produce. Also, holding returns are specific to $P$ and $M$, and $\rho_P \leq 0$ and $\rho_M \leq 0$, with $-\rho_P$ and $-\rho_M$ interpreted as storage costs.

Let $\alpha_{ij}$ be the rate at which $i$ meets $j$ (subject to regularity conditions, there is always a population $\mathbf{n}$ consistent with this). In $PC$ matches, $P$ gives the indivisible good to $C$ for $q_{CP}$. In $MC$ matches, if $M$ has $a = 1$ he gives it to $C$ for $q_{CM}$. In $MP$ matches, they cannot trade if $a = 1$, and may or may not trade if $a = 0$, but if they do $M$ transfers $q_{MP}$ to $P$. Let $m$ be the measure of type $M$ with $a = 1$. The $q_i$ that $i$ gives $j$ splits the surplus, where $\theta_{ij}$ is the share or bargaining power of $i$, with $\theta_{ji} + \theta_{ij} = 1$, consistent with Nash or Kalai bargaining since $u(q) = c(q) = q$. We need to determine the fraction of $P$ that produce, $\varepsilon$, and the fraction of $M$ that

interactions between money and credit or other assets. Wallace (1997) and Katzman et al. (2003) study money-output correlations. Johri (1999) further analyzes dynamics. Wallace and Zhou (1997) discuss currency shortages, Wallace (2000) and Lee et al. (2005) discuss maturity and denomination structures, while Ales et al. (2008) and Redish and Weber (2011) discuss related issues in economic history. Williamson (1999) adds inside money. Several papers (see Section 11) add private information. Burdett et al. (2014) study price dispersion. International applications include Curtis and Waller (2000,2003), Wright and Trejos (2001), Camera et al. (2004) and Craig and Waller (2004).

[24] A simplifying assumption here is that all agents stay in the market forever; the original model had $M$ stay, while $P$ and $C$ exit after a trade, to be replaced by clones. Nosal et al. (2013) nest these by letting agents stay with some probability after a trade.

actively trade, $\tau$ (similar to entry in Pissarides 2000). We concentrate on stationary, pure-strategy, asymmetric equilibria, where a fraction $\varepsilon$ of type $P$ and a fraction $\tau$ of type $M$ are always active while the rest sit out. A key economic observation is that since storage costs are sunk when traders meet, there are holdup problems.[25]

Suppose to illustrate the method that $\tau = 0$, so $C$ can only trade directly with $P$. Then $rV_C = \alpha_{CP}\varepsilon (u - q_{CP})$, where $u - q_{CP}$ is $C$'s surplus, since the continuation value $V_C$ cancels with his outside option $V_C$. Notice $\varepsilon$ appears because we assume uniform random matching, in the sense that $C$ can contact $P$ even if the latter is not participating (imagine contacts occurring by phone, with $\alpha_{CP}$ the probability per unit time $C$ and $P$ are connected, but $P$ only picks up if he is on the market). Bargaining implies $u - q_{CP} = \theta_{CP}u$, where $u$ is total match surplus since for $P$ the continuation value also cancels with his outside option. Then $rV_C = \alpha_{CP}\varepsilon\theta_{CP}u$. Similar expressions hold for $\tau \neq 0$, and for $V_P$ and $V_a$, where the latter is $M$'s value function when he has $a \in \{0, 1\}$. In fact, from these DP equations, one might observe this model looks a lot like the one in Section 4, where the indivisible good was money, except here $u(q) = c(q) = q$. We said earlier $q$ can be interpreted as a payment instrument, but one can also call the indivisible good money, since it is a storable object acquired by $M$ and used to get $q$ from $C$. For the record, Rubinstein and Wolinksy (1987) call $q$ money as a synonym for transferable utility – but see Wright and Wong (2013) for an extended discussion of this dubious practice.

In any case, the BR conditions are standard, e.g., $\varepsilon = 1$ if $V_P > 0$, $\tau = 1$ if $\Delta > 0$, etc. So is the SS condition. Equilibrium satisfies the obvious conditions. From this we get the transfers in direct trade $q_{CP} = \theta_{PC}u$, wholesale trade $q_{MP} = \theta_{PM}\Delta$, and retail trade $q_{CM} = \theta_{MC}u + \theta_{CM}\Delta$. The spread, or dealer markup, is $q_{CM} - q_{MP} = \theta_{MC}u + (\theta_{MC} - \theta_{MP})\Delta$. Equilibrium exists and is generically unique, as shown in Figure 5 in $(-\rho_P, -\rho_M)$ space. When $-\rho_P$ is high we get $\varepsilon = 0$, so the

[25] In the original Rubinstein-Wolinsky model $\rho_j = 0$, but there is still a holdup problem because the transfer from $M$ to $P$ is sunk when $M$ meets $C$. Given this, they discuss a consignment arrangement, where $M$ transfers $q$ to $P$ only after trading with $C$, so it is not sunk when bargaining with $C$. This is a example of trading institutions arising endogenously in response to frictions, although it may or not be feasible, depending on the physical environment (it obviously does not work if $M$ and $P$ cannot reconvene after trading, or if they can but $M$ cannot commit). In any case, while it may alter payoffs, none of this affects efficiency in the original Rubinstein-Wolinsky setting (see below). By contrast, the storage costs here deplete real resources – they are not just transfers – and are necessarily sunk when $P$ or $M$ meet $C$.

market is closed. When $-\rho_P$ is low and $-\rho_M$ high we get $\varepsilon = 1$ and $\tau = 0$, so there is production but no intermediation. When both are low we get intermediation. For some parameters, $P$ enters with probability $\varepsilon \in (0,1)$, with $m$ adjusting endogenously to make $V_P = 0$. This is related to discussions of search externalities in the literature: here entry by $P$ does not affect the rate at which other $P$'s meet counterparties, it increases $m$, but this still makes it harder for $P$ to trade. Notice also that when $P$ has a poor storage technology, a low rate of finding $C$, or low bargaining power, *intermediation is essential*: the market opens iff middlemen are active.

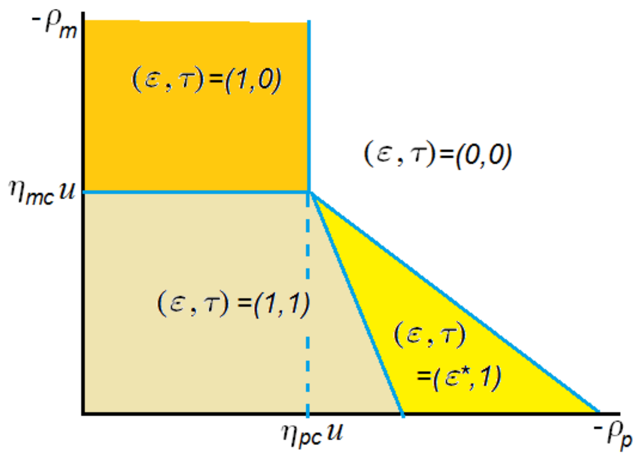


Figure 5: Equilibria with Middlemen

Rubinstein and Wolinsky (1987) use $\theta_{ij} = 1/2\ \forall ij$ and $\rho_P = \rho_M = 0$. In that case, $P$ is always active, and $M$ is active iff $\alpha_{MC} > \alpha_{PC}$, as is socially efficient. More generally, again related to Mortensen (1982) and Hosios (1990), in Nosal et al. (2013) equilibrium is efficient for all values of other parameters iff $\theta_{PC} = 1$, $\theta_{MC} = 1$ and $\theta_{PM} = \theta^*_{PM} \in (0,1)$. For other $\theta$'s, $\varepsilon$ can be too high or too low, while $\tau$ can be too low but not too high. Heuristically, $\varepsilon$ or $\tau$ are too low when $P$ or $M$ have low bargaining power, making holdup problems severe, and $\varepsilon$ can be too high due to the above-mentioned search externality. One can say more, and there are other papers on middlemen, including a few closely related to this survey (e.g., Li 1998, Schevchenko 2004 and Masters 2007,2008). To save space, we refer to Wright and Wong (2014) for a longer list of citations, and turn to another application.

# 6 Intermediation in Financial Markets

Duffie et al. (2005) propose a search-and-bargaining model of OTC markets that provides a natural way to think about standard measures of liquidity, like bid-ask spreads, execution delays and trading volume. Their model is complementary to information- or inventory-based models. It is also realistic.[26]

The baseline model has agents called $I$ and $D$, for investors and dealers. There is a fixed supply $A$ of an indivisible asset, with $a \in \{0, 1\}$ denoting the asset position of $I$. In the simplest formulation, $D$ does not hold the asset (but see Weill 2007,2008). There is potential bilateral trade when $I$ meets either another $I$ or $D$. In addition, $D$ has continuous direct access to a competitive interdealer market. Gains from trade emerge because $I$'s valuation of the dividend is subject to idiosyncratic shocks, with several interpretations discussed in Duffie et al. (2007). The flow utility for $I$ with $a = 1$ and valuation $j$ is $\rho_j$, where $j \in \{0, 1\}$ and $\rho_1 > \rho_0$, and valuations switch as follows: in any state $j$, there are shocks, with $j' = 1$ arriving at Poisson rate $\omega_1$ and $j' = 0$ at rate $\omega_0$ (see Gavazza 2011 for an application with continuum of valuations).

As in Section 5, there is a divisible good anyone can consume and produce, for $u(q) = q$ and $c(q) = q$. It is yet to be determined if $I$ trades with $D$, but $I$ trades with another $I$ iff one has $a = 1$ and $j = 0$ while the other has $a = 0$ and $j = 1$ (a double coincidence). Let $V_{aj}$ be $I$'s value function with asset position $a$ and valuation $j$, so that $\Delta_j = V_{1j} - V_{0j}$ is the value to $I$ of acquiring the asset when he is in state $j$. When $I$ trades with $I$, the total surplus is $S_I = \Delta_1 - \Delta_0$ and the one that gives up the asset gets a transfer $q_I$ (the subscript indicates $I$ trades with $I$). This transfer yields the $I$ that entered with asset position $a$ a share of $S_I$ given by $\theta_a$, with $\theta_0 + \theta_1 = 1$, as in Section 4. Hence, the individual gains from trade are $q_I - \Delta_0 = \theta_1 S_I$ and $\Delta_1 - q_I = \theta_0 S_I$, and hence $q_I = \theta_0 \Delta_0 + \theta_1 \Delta_1$.

The rate at which $I$ meets $I$ is $\alpha_I$, and the probability that $I$ has asset position $a$ and valuation $j$ is $m_{aj}$. The rate at which $I$ meets $D$ is $\alpha_D$. When $I$ meets $D$, if they trade, one can think of the latter as trading in the interdealer market on behalf

[26] "Many assets, such as mortgage-backed securities, corporate bonds, government bonds, US federal funds, emerging-market debt, bank loans, swaps and many other derivatives, private equity, and real estate, are traded in ... [over-the-counter] markets. Traders in these markets search for counterparties, incurring opportunity or other costs. When counterparties meet, their bilateral relationship is strategic; prices are set through a bargaining process that reflects each investor's alternatives to immediate trade." (Duffie et al. 2007).

of the former for a fee. Obviously this is only relevant when $D$ meets $I$ with $a = 0$ and $j = 1$ or meets $I$ with $a = 1$ and $j = 0$, since these are the only type $I$ agents that currently want to trade. If $I$ gets the asset, $D$ gets $q_A$ (for ask); if $I$ gives up an asset, $D$ gets $q_B$ (for bid). Since the cost to $D$ of getting an asset on the interdealer market is $q_D$, when $D$ gives an asset to $I$ in exchange for $q_A$, the bilateral surplus is $S_A = \Delta_1 - q_D$. And when $D$ gets an asset from $I$ the surplus is $S_B = q_D - \Delta_0$. If $\theta_D$ is $D$'s bargaining power when he deals with $I$, then

$$q_A = \theta_D \Delta_1 + (1 - \theta_D) q_D \text{ and } q_B = \theta_D \Delta_0 + (1 - \theta_D) q_D. \tag{19}$$

One can interpret $q_A - q_D$ as the fee $D$ charges when he gets an asset for $I$ in the interdealer market, and similarly for $q_D - q_B$. The round-trip spread, related to the markup in Section 5 is $s = q_A - q_B = \theta_D (\Delta_1 - \Delta_0) > 0$.

Trading strategies are summarized by $\boldsymbol{\tau} = (\tau_A, \tau_B)$, where $\tau_A$ is the probability when $D$ meets $I$ with $a = 0$ and $j = 1$ they agree to exchange the asset for $q_A$, while $\tau_B$ is the probability when $D$ meets $I$ with $a = 1$ and $j = 0$ they agree to exchange the asset for $q_B$. The BR conditions are again standard, e.g. $\tau_A = 1$ if $\Delta_1 > q_D$, etc. The reason $I$ with $a = 0$ and $j = 1$ might not trade when he meets $D$ is that $q_D \geq \Delta_1$ is possible. Similarly, the reason $I$ with $a = 1$ and $j = 0$ might not trade when he meets $D$ is that $q_D \leq \Delta_0$. For market clearing, because the asset is indivisible, it will be typically the case that $q_D \in \{\Delta_0, \Delta_1\}$, and the long side of the market is indifferent to trade. If the measure of $D$ trying to acquire an asset exceeds the measure trying to off-load one, then $q_A = q_D = \Delta_1$ and $\tau_A \in (0, 1)$. In the opposite case, $q_B = q_D = \Delta_0$ and $\tau_B \in (0, 1)$. Since the measure of $I$ trying to acquire assets is $m_{10}$ and the measure trying to divest themselves of assets it $m_{01}$, the former is on the short side iff $m_{10} < m_{01}$ iff $A < \hat{A} \equiv \omega_1 / (\omega_0 + \omega_1)$.

The SS and DP equations are standard – e.g., the flow payoff for $I$ with $a = 1$ and low valuation is the dividend, plus the expected value of trading with $I$ or $D$, plus the capital gain from a preference shock:

$$rV_{10} = \rho_0 + \alpha_I m_{01} \theta_1 S + \alpha_D \tau_B (q_B - \Delta_0) + \omega_1 (V_{11} - V_{10}).$$

An equilibrium is a list $\langle \mathbf{V}, \boldsymbol{\tau}, \mathbf{m} \rangle$ satisfying the usual conditions, and it exists uniquely. The terms of trade are are easily recovered, as are variables like the bid-ask spread, $q_A - q_B = \theta_D (\rho_1 - \rho_0) / \Sigma$, where $\Sigma > 0$. This stylized structure, with a core of highly-connected dealers and a periphery of investors who trade with each other or with dealers, is a reasonable picture of many OTC markets. It is also flexible, given the proportion of trade that is intermediated is $\alpha_D/(\alpha_D+\alpha_I)$.

If $\alpha_D$ is small, most exchange occurs between individual investors, as is the case for specialized derivatives or Fed Funds. A small $\alpha_I$ better approximates trading on NASDAQ. The case $\alpha_D = 0$ is interesting in terms of making connections between the money and finance literatures: the model here has gains from trading $a$ due to heterogeneous valuations, with $q$ serving as a payment instrument; the one in Section 4 has gains from trading $q$ with $a$ serving as a payment instrument (see Trejos and Wright 2013 for a discussion). From the point of view of intermediation theory, $\alpha_I = 0$ is interesting, and also nice because the value functions and terms of trade are independent of $m$. In this case one can easily show that spreads are decreasing in $\alpha_D$ and increasing $\theta_D$. When $r \to 0$, $q_A$, $q_B$ and $q_D$ all converge to the same limit, which is $\rho_0/r$ if $A > \hat{A}$ and $\rho_1/r$ if $A < \hat{A}$. There are also implications for trade volume, something financial analysts often associate with liquidity, although the results can be sensitive to the restriction $a \in \{0,1\}$. The time has come to relax this restriction, first in monetary theory, then in models of asset markets.

# 7 The Next Generation

Here we generalize $a \in \{0,1\}$ to $a \in \mathcal{A}$ for some less restrictive $\mathcal{A}$. One option is $\mathcal{A} = \{0, 1...\bar{a}\}$, where $\bar{a}$ may be finite or infinite, and proceeds with a combination of analytic and computational methods.[27] The main difficulty is the endogenous distribution of assets across agents, $F(a)$, but some results are available. Under certain assumptions, $F$ is geometric if $\bar{a} = \infty$, and truncated geometric if $\bar{a} < \infty$. One can also endogenize $\bar{a}$, and for $\bar{a} < \infty$ and fiat money, one can show the optimum quantity is $A^* = \bar{a}/2$, generalizing Section 3 where $\bar{a} = 1$ and $A^* = 1/2$.

Molico (2006) studies the case of fiat currency with $\mathcal{A} = [0, \infty)$, allowing $A_{+1} = (1+\pi)A$, where the subscript indicates next period, and $\pi$ is the rate of monetary

[27] See Camera and Corbae (1999), Rocheteau (2000), Berentsen (2002), Zhu (2003) and Deviatov and Wallace (2009). Following Green and Zhou (1998,2002), see also Zhou (1999,2003), Kamiya and Sato (2004), Kamiya et al. (2005), and Kamiya and Shimizu (2006,2007$a$,$b$). Interestingly, Green-Zhou models can have a continuum of stationary equilibria. For an illustration in the context of the benchmark model discussed below, and a refinement, see Jean et al. (2010).

expansion generated by a lump-sum transfer $T$.[28] As above, an agent can be a buyer or a seller, depending on who he meets, but now he can buy whenever $a > 0$. We maintain commitment and information assumptions precluding credit, and set $\delta = 0$ to eliminate barter. Then let $q(a_B, a_S)$ be the quantity of output and $d(a_B, a_S)$ the amount of dollars traded when the buyer has $a_B$ and the seller $a_S$, assuming for simplicity $(a_B, a_S)$ is observed by both. Then

$$
\begin{aligned}
V(a) \;=\; & (1-2\alpha\sigma)\beta V_{+1}(a+T) \\
& +\alpha\sigma \int \{u[q(a, a_S)] + \beta V_{+1}[a - d(a, a_S) + T]\} \, dF(a_S) \\
& +\alpha\sigma \int \{-c[q(a_B, a)] + \beta V_{+1}[a + d(a_B, a) + T]\} \, dF(a_B),
\end{aligned} \tag{20}
$$

where $V(a)$ is the value function of an agent with $a$ assets. The first term on the RHS is the expected value of not trading; the second is the value of buying from a random seller; and the third is the value of selling to a random buyer.

To determine terms of trade, consider generalized Nash bargaining,

$$
\max_{q,d} S_B(q, d, a_B, a_S)^{\theta} \, S_S(q, d, a_B, a_S)^{1-\theta}, \tag{21}
$$

where $S_B(\cdot) = u(q) + \beta V_{+1}(a_B + T - d) - \beta V_{+1}(a_B + T)$ and $S_S(\cdot) = -c(q) + \beta V_{+1}(a_S + d + T) - \beta V_{+1}(a_S + T)$ are the surpluses. The maximization is subject to $q \geq 0$ and $-a_S \leq d \leq a_B$, which might *look* similar to what one sees in CIA models, but they are simply feasibility constraints, saying agents cannot hand over more than they have (as above, credit is unavailable due to commitment and information frictions). There is a law of motion for $F(a)$ with a standard ergodicity condition. A stationary equilibrium is a list $\langle V, q, d, F \rangle$ satisfying the obvious conditions.

Molico (2006) numerically analyzes the relationship between inflation and dispersion in the price, $p(\cdot) = d(\cdot)/q(\cdot)$, and asks what happens as frictions decrease. He also studies the welfare effects of inflation. Increasing $A$ by giving agents transfers proportional to their current $a$ has no real effect – it is like a change in units. But a lump-sum transfer $T > 0$ compresses the distribution of real balances, since when the value of a dollar falls it hurts those with low $a$ less than those with high $a$. Since

[28] This may be less relevant for real assets, but one can let stock of these assets change, say, by having them go bad with some probability, with new ones created to keep $A$ constant. Li (1994,1995), Cuadras-Morato (1997), Deviatov (2006) and Deviatov and Wallace (2014) do something like this with fiat money to proxy for inflation in first- or second-generation models.

those with low $a$ don't buy very much, and those with high $a$ don't sell very much, this compression is good for economic activity: expansionary monetary policy helps by spreading liquidity around. At the same time, inflation detrimentally reduces real balances, and policy must balance these countervailing effects. As Wallace (2014) conjectures, some inflation may well desirable in any economy with such a tradeoff.

Following along these lines, Jin and Zhu (2014) study dynamic transitions for $F(a)$ after various types of monetary injections, and show how the redistributional impact can have rather interesting effects on output and prices. They emphasize a different effect than Molico, whereby output in a match $q(a_B, a_S)$ is decreasing and convex in $a_S$, and hence, a policy that increases dispersion in real balances increases average $q$. This leads to slow increases in the aggregate price level after monetary injections. The reason is not that prices are sticky – indeed, $q$ and $d$ are are determined by bargaining in each trade – but that the increase in output keeps the nominal price level from rising too quickly during the transition. This is very important because it shows how Keynesian implications do not follow from time-series observations that money shocks affect mainly output and not prices in the short run.

A different modeling approach when $\mathcal{A} = [0, \infty)$ tries to harness the distribution $F(a)$ somehow. One method due to Shi (1997$a$), assumes a continuum of households, each with a continuum of members, to get a degenerate distribution across households. The decision-making units are families, whose members search randomly, as above, but at the end of each trading round they return home and share the proceeds. By a law of large numbers, each family starts the next trading round with the same $a$. The approach is discussed at length by Shi (2006).[29] Another method, due to Menzio et al. (2012) and Sun (2012), uses directed search and free entry, so that while there is a distribution $F(a)$, the market segments into submarkets in such a way that agents do not need to know $F(a)$.

Here we take a different path, following Lagos and Wright (2005), which integrates search-based models like those presented above and standard frictionless models. One advantage is that this reduces the gap between monetary theory with some claim to

[29] In addition to work cited below, see Head and Kumar (2005), Head et al. (2010), Head and Shi (2003), Kumar (2008), Peterson and Shi (2004) and Shi (2001,2005,2008,2014). Rauch (2000), Berentsen and Rocheteau (2003$b$) and Zhu (2008) discuss a technical issue that comes up in large-family models; this is avoided by the models discussed below.

microfoundations and mainstream macro. That this gap was big is all too clear from Azariadis (1993): "Capturing the transactions motive for holding money balances in a compact and logically appealing manner has turned out to be an enormously complicated task. Logically coherent models such as those proposed by Diamond (1982) and Kiyotaki and Wright (1989) tend to be so removed from neoclassical growth theory as to seriously hinder the job of integrating rigorous monetary theory with the rest of macroeconomics." Or, as Kiyotaki and Moore (2001) put it, "The matching models are without doubt ingenious and beautiful. But it is quite hard to integrate them with the rest of macroeconomic theory – not least because they jettison the basic tool of our trade, competitive markets."

The Lagos-Wright (2005) model brings some competitive markets back on board, to continue the nautical metaphor. But it also maintains some frictions from the models presented above – one doesn't want to go too far, since, as Helwig (1993) emphasized, the "main obstacle" in developing a framework for studying monetary systems is "our habit of thinking in terms of frictionless, organized, i.e. Walrasian markets." Also, realistically, one wants some of both in the same setting. Much activity in our economic lives is quite centralized (it is easy to trade, we take prices as given etc.) as in GE theory, but there is also much that is decentralized (it is not easy to find counterparties, it can be hard to get credit etc.) as in search theory. Although one might imagine different ways to combine markets with and without frictions, Lagos and Wright try dividing each period into two subperiods: in the first, agents interact in a decentralized market, or DM; in the second, they interact in a frictionless centralized market, or CM.[30]

We now work through the details of Lagos-Wright, which has become a workhorse in the literature. DM consumption is still denoted $q$, while CM consumption is a different good $x$ (we can have a vector of CM goods $\mathbf{x}$ without changing any key

[30] Some authors call the subperiods day and night as a mnemonic device; we avoid this because it seems to bother a few people – maybe they don't like circadian models? More substantively, we continue to impose the frictions that make money essential, including imperfect commitment and information. It is important to recognize that the presence of a CM does not logically imply agents can communicate DM deviations to the population at large, even if one may get that impression from Aliprantis et al. (2006,2007$a$). To clarify their contribution, they give agents the ability to communicate DM deviations when they reconvene in the CM, which is is a legitimate assumption, but so is the assumption that they cannot communicate deviations. Moreover, even given their formulation, in Aliprantis et al. (2007$b$) they show how to ensure money is still essential. There is more to be done on these issues; see Araujo et al. (2012) for recent developments.

results). For now, $x$ is produced one-for-one using labor $\ell$, so the CM real wage is 1. As above, in the DM, households can end up as buyers or sellers depending on who they meet. In the former case their period utility is $\mathcal{U}(x, 1-\ell) + u(q)$, while in the latter case it is $\mathcal{U}(x, 1-\ell) - c(q)$, where $\mathcal{U}(\cdot)$ is monotone and concave while $u(\cdot)$ and $c(\cdot)$ are as in Section 4. For ease of presentation, let us begin with $\mathcal{U}(x, 1-\ell) = U(x) - \ell$ and discuss alternatives later. In the DM, the value function $V(\cdot)$ is described by (20) with one change: wherever $\beta V_{+1}(\cdot)$ appears on the RHS, replace it with $W(\cdot)$, since before going to the next DM agents now visit the CM, where $W(\cdot)$ is the value function. It satisfies

$$W(a) = \max_{x,\ell,\hat{a}} \{U(x) - \ell + \beta V_{+1}(\hat{a})\} \text{ st } x = \phi(a - \hat{a}) + \rho a + \ell + O,$$

where $a$ and $\hat{a}$ are asset holdings when trading opens and closes, $\phi$ is the price of $a$ in terms of $x$, $\rho$ is a dividend, and $O$ denotes any other purchasing power.[31]

There are constraints $x \geq 0$, $\hat{a} \geq 0$ and $\ell \in [0, 1]$ that we ignore for now. Then, eliminating $\ell$, we have

$$W(a) = (\phi + \rho) a + O + \max_{x} \{U(x) - x\} + \max_{\hat{a}} \{-\phi\hat{a} + \beta V_{+1}(\hat{a})\}. \tag{22}$$

Several results are immediate: (i) $x = x^*$ is pinned down by $U'(x^*) = 1$; (ii) $W(a)$ is linear with slope $\phi + \rho$; and (iii) $\hat{a}$ is independent of wealth. By (iii) we get history independence ($\hat{a}$ is orthogonal to $a$), and hence $F(\hat{a})$ is degenerate when there is a unique maximizer $\hat{a}$. In most applications there is a unique such $\hat{a}$. In some, like Galenianos and Kircher (2008) or Dutu et al. (2012), $F$ is nondegenerate for endogenous reasons, but the analysis is still tractable due to history independence. Similarly, with exogenous heterogeneity (see below), $F$ is only degenerate after conditioning on type, but that is enough for tractability.

It is important to know these results actually hold for a larger class of specifications. Instead of quasi-linear utility, we can assume $\mathcal{U}(x, 1-\ell)$ is homogeneous of degree 1, as a special case of Wong (2012), and hence we can use common preferences like $x^\gamma (1-\ell)^\gamma$ or $[x^\gamma + (1-\ell)^\gamma]^{1/\gamma}$. Alternatively, as shown in Rocheteau et al. (2008), the main results all go through with *any* monotone and concave $\mathcal{U}(x, 1-\ell)$

[31] This includes transfers net of taxes. Note also that instead of injecting new money via transfer $T$, in this model, government can buy CM goods and nothing changes but $\ell$. We mention this in case one worries about the realism of lump-sum money transfers.

if we assume indivisible labor, $\ell \in \{0,1\}$, and use employment lotteries as in Hansen (1985) or Rogerson (1988). Any of these assumptions, $\mathcal{U}$ quasi-linear or homogeneous of degree 1, or $\ell \in \{0,1\}$, imply $\hat{a}$ is independent of $a$. There is one caveat: we need an interior solution at least in some periods, but that does not seem too extreme.

We now move to the DM, characterized by search and bargaining. Here it makes a difference whether $a$ is a real or fiat object. One reason is that we know the constraint $d \leq a_B$, which again says agents cannot turn over more than they have, binds with fiat money, but we cannot be sure with real assets. So for now consider $\rho = 0$. Using $d = a_B$, the generalized Nash bargaining solution is

$$\max_q \left[u(q) - \phi a_B\right]^\theta \left[\phi a_B - c(q)\right]^{1-\theta}. \tag{23}$$

The simplicity of (23) is due to $W'(a) = \phi$ being independent of $a_B$ or $a_S$ (which, by the way, means agents do not need to observe each other's $a$ to know their marginal valuations). Indeed, (23) is the same as Nash bargaining in Section 4, except the value of the monetary payment is $\phi a_B$ instead of $\Delta$. Hence, the outcome is $\phi a_B = v(q)$ instead of $\Delta = v(q)$, but $v(q)$ is still given by (18).

Moreover, we can now easily use a variety of alternative solution concepts. For Kalai bargaining, simply take $v(q)$ from (17) instead of (18).[32] For Walrasian pricing, take $v(q) = Pq$, where $P$ is the price of DM goods in terms of $x$, over which individuals have no influence even though $P = c'(q)$ in equilibrium.[33] Here we keep the mechanism $v(q)$ general, imposing only $v'(\cdot) > 0$ plus one other condition: $\phi a_B \geq v(q^*)$ implies a buyer pays $d = v(q^*)/\phi$ and gets $q^*$; and $\phi a_B < v(q^*)$ implies he pays $d = a_B$ and gets $q = v^{-1}(\phi a_B)$ (one can show this holds automatically for competitive pricing, standard bargaining solutions and many other mechanisms).

[32] Aruoba et al. (2007) advocate Kalai bargaining in the model because: (i) it makes buyers' surplus increasing in $a$, which is not always true for Nash; (ii) this eliminates incentives to hide assets; (iii) it makes $V(a)$ concave; and (iv) it is generally much more tractable. Some of these issues with Nash can be handled by letting agents bring less than all their assets to the bargaining table (to the DM), as in Geromichalos et al. (2007), Lagos and Rocheteau (2008) and Lagos (2010b). Morever, as Kalai (1977) himself says, his solution involves interpersonal utility comparisons. Based on all this, it seems good to remain agnostic about bargaining solutions.

[33] To motivate this, Rocheteau and Wright (2005) assume agents meet in large groups in the DM, not bilaterally, while maintaining the frictions making money essential. One can make the DM look even more Walrasian by setting $\alpha\sigma = 1$. However, $\alpha\sigma < 1$ is nice because it captures a *precautionary* demand for liquidity, described by Keynes (1936) as providing for "contingencies requiring sudden expenditure and for unforeseen opportunities of advantage." Also, $\alpha\sigma < 1$ avoids the problem in CIA models where velocity must be 1 (see Telyukova and Visschers 2013).

For any such $v(q)$, the DM value function satisfies

$$V(a) = W(a) + \alpha\sigma \{u[q(a)] - \phi a\} + \alpha\sigma \int \{\phi\tilde{a} - c[q(\tilde{a})]\} dF(\tilde{a}), \tag{24}$$

where $\hat{a}$ is money held by an individual while $\tilde{a}$ is held by others, which we allow to be random at this stage, although in equilibrium $\tilde{a} = A$. Then one can easily derive

$$V'(a) = \phi + \alpha\sigma \{u'[q(a)]q'(a) - \phi\} = \phi \left\{1 + \alpha\sigma \frac{u'[q(a)]}{v'[q(a)]} - \alpha\sigma\right\},$$

using $q'(a) = \phi/v'(q)$, which follows from $\phi a = v(q)$. We now insert this into the FOC from the previous CM, $\phi_{-1} = \beta V'(a)$, where the $-1$ subscript indicates last period. The result is the Euler equation

$$\phi_{-1} = \beta\phi [1 + \alpha\sigma\lambda(q)], \tag{25}$$

where $\lambda(q) \equiv u'(q)/v'(q) - 1$ is the *liquidity premium*, or equivalently, the *Lagrange multiplier* on the constraint in the problem $\max \{u(q) - v(q)\}$ st $v(q) \leq a\phi$.

Although we focus mainly on stationary equilibrium, for the moment let's proceed more generally. Using $\phi a = v(q)$, $a = A$, and $A = (1+\pi) A_{-1}$, (25) becomes

$$(1+\pi) v(q_{-1}) = \beta v(q) [1 + \alpha\sigma\lambda(q)]. \tag{26}$$

Given any path for $\pi$, one can study dynamics as in Section 4. Various authors have shown there can be cyclic, chaotic and stochastic equilibria, again due to the self-referential nature of liquidity (Lagos and Wright 2003; Ferraris and Watanabe 2011; Rocheteau and Wright 2013; Lagos and Zhang 2013; He et al. 2013).

In a stationary equilibrium $z = \phi A$ is constant, so $\phi/\phi_{+1} = 1 + \pi$ is the gross inflation rate – a version of the Quantity Theory. To present the results compactly, let $\iota$ be the interest rate on an illiquid nominal bond, i.e., one that cannot be traded in the DM for whatever reason, or simply as an accounting exercise asking how much money you would pay in the CM to get a dollar in the next CM. The logic behind the Fisher equation implies $1 + \iota = (1+\pi)(1+r)$, where $r$ is the interest rate on an illiquid real bond. So policy can be described by $\iota$, and (25) reduces to

$$\iota = \alpha\sigma\lambda(q). \tag{27}$$

Given $\iota$, (27) determines $q$. Real balances are $z = v(q)$ and the price level is $1/\phi = A/v(q)$ – another version of the Quantity Theory. The FOC from the CM determines $x$, and $\ell$ comes from the budget equation to complete equilibrium.[34] There is a

[34] For individuals, CM labor depends on $a$, say $\ell(a)$, but in aggregate $\ell = \int \ell(a) = x$.

unique such equilibrium if $\lambda'(q) < 0$. For many mechanisms, but not all, $\lambda'(q) < 0$ is automatic (e.g., Kalai and Walras, but not generalized Nash). However, even without $\lambda'(q) < 0$ Wright (2010) shows that there is generically a unique stationary monetary equilibrium, with natural properties like $\partial q/\partial \iota < 0$, $\partial q/\partial \theta > 0$ etc.

As for efficiency, for generalized Nash bargaining, Lagos and Wright (2005) show $q = q^*$ under two conditions (see also Berentsen et al. 2007). The first is the Friedman rule $\iota = 0$, which eliminates the usual intertemporal wedge in money demand. The second is $\theta = 1$, again related to Mortensen (1982) and Hosios (1990). Heuristically, when agents bring money to the DM they are making an investment in liquidity, and they underinvest if they do not get the appropriate return. With Nash bargaining this means $\theta = 1$. If $\theta < 1$ is immutable, it would seem desirable to set $\iota < 0$, but that is not feasible, since there is no equilibrium with $\iota < 0$. This is a New Monetarist version of the *zero-lower-bound problem* currently emphasized in New Keynesian circles. Although here it has nothing to do with nominal rigidities, it can still be ascribed to a poor pricing mechanism – e.g., Kalai bargaining with any $\theta > 0$ delivers $q^*$ at $\iota = 0$ (Aruoba et al. 2007).[35] We return to this in Section 9.

That's the basic model. While one can relax any of our assumptions, something like quasi-linearity or the options mentioned above is needed for history independence. Without that, the model is well posed and (even more) interesting, but requires numerical methods, as in Chiu and Molico (2010,2011). By analogy, while specifications with heterogeneity and incomplete markets are worth studying computationally in mainstream macro, it is nice to have standard growth theory as a benchmark to analyze existence, uniqueness or multiplicity, dynamics, efficiency, etc. For monetary economics, the framework just presented is our benchmark.

# 8 Extending the Benchmark

Some versions of our baseline environment have the CM and DM open simultaneously with agents transiting randomly between them (Williamson 2007); others have multiple rounds of DM trade between CM meetings or vice-versa (Berentsen et al. 2005;

[35] One can also design a mechanism $v(\cdot)$ that for some parameters delivers $q^*$ even at $\iota > 0$, as in Hu et al. (2009). This is relevant because $\iota = 0$ requires deflation, $\pi = \beta - 1 < 0$, which typically requires taxation. As Andolfatto (2013) argues, the assumptions that make money essential can also make it difficult to enforce taxes.

Telyukova and Wright 2008; Ennis 2009); others use continuous time (Craig and Rocheteau 2008*b*; Rocheteau and Rodriguez 2013). An extension in Lagos and Rocheteau (2005) and Rocheteau and Wright (2005), on which we spend more time, has two distinct types, called buyers and sellers because in every DM the former want to consume but cannot produce while the latter produce but do not consume. One cannot do this with only DM trade – why work for money if there is no occasion to spend it? But here sellers produce in the DM and spend the receipts in the CM. This makes the DM a two-sided market, with distinct ex ante types, buyers and sellers, as opposed to traders that can be either depending on meetings, which makes it more natural to incorporate general matching technologies and participation decisions.[36]

A two-sided market also makes it natural to consider competitive search, where market makers set up submarkets in the DM to attract buyers and sellers and charge them entrance fees.[37] Normalize the measure of buyers to 1 and let $n$ be the measure of sellers. In general, they meet in the DM according to a standard matching technology, where $\alpha(n)$ is the probability a buyer meets a seller and $\alpha(n)/n$ is the probability a seller meets a buyer. A submarket involves posting $(q, z, n)$ in the CM, describing the next DM by the terms of trade – buyers and sellers commit to swapping $q$ units of output for $z = \phi a$ real balances if they meet – as well as the seller-buyer ratio $n$ that agents use to compute the probability of meeting. Market makers design $(q, z, n)$ to maximize buyer's surplus subject to sellers getting a minimal surplus, or vice-versa. In equilibrium, surpluses are dictated by the market.

Algebra reduces the market maker problem to

$$\max_{q,z,n} \{\alpha(n)[u(q) - z] - \iota z\} \text{ st } \alpha(n)[z - c(q)] = n\bar{S}_S,$$

given the seller's surplus $\bar{S}_S$. Eliminating $z$ and taking the FOC's, we get

$$\begin{aligned} \alpha(n) u'(q) &= [\alpha(n) + \iota] c'(q) && (28) \\ \alpha'(n)[u(q) - c(q)] &= \bar{S}_S \left[1 + \frac{\iota(1-\epsilon)}{\alpha(n)}\right], && (29) \end{aligned}$$

[36] By analogy, Pissarides (2000) is a two-sided market with workers and firms; Diamond (1982) is a one-sided market. The former is able to use matching functions with arrival rates depending on market tightness and entry, which is interesting even with constant returns; the latter is not.

[37] In equilibrium, the fee is 0. This market maker story comes from in Moen (1997) and Mortensen and Wright (2002). We can instead let sellers post terms of trade to attract buyers or vice-versa; sometimes these are equivalent, sometimes not (Faig and Huangfu 2007). Monetary models with competitive search, in addition to those discussed below, include Faig and Jerez (2006), Huangfu (2009), Dong (2011), Dutu et al. (2011) and Bethune et al. (2014).

where $\epsilon = n\alpha'(n)/\alpha(n)$ is the elasticity of the matching function wrt participation by sellers. One can assume all agents participate in the DM for free, so that $n$ is given. Then (28) determines $q$, while (29) determines $\bar{S}_S$ and hence also $\bar{S}_B$. Or one can assume sellers have a cost $\kappa$ to enter the DM payable in the next CM. Then $\bar{S}_S = \kappa$, and (28)-(29) determine $q$ and $n$ jointly. In either case, $\iota = 0$ implies $q = q^*$, and given this, with endogenous entry, it implies $\alpha'(n^*)[u(q^*) - c(q^*)] = \kappa$.[38]

Lagos and Rocheteau (2005) fix $n$ but endogenize buyers' search intensity. They first show that with bargaining the time it takes buyers to spend their money increases with $\pi$, counter to the well-known "hot potato" effect of inflation (e.g., see Keynes 1924, p. 51). This is because $\pi$, as a tax on money, reduces the DM surplus and hence search. But with competitive posting, although $\pi$ lowers the total surplus, for some parameters it raises buyers' share. Hence, they can get buyers to spend money faster at higher $\pi$ by having them increase search effort. Alternatively, Liu et al. (2011) get buyers to spend their money faster at higher $\pi$, even with bargaining, by giving buyers rather than sellers a participation decision. In this case, $\pi$ makes them match faster by reducing buyer entry, given a standard matching technology. Alternatively, in Nosal (2011) $\pi$ makes buyers spend faster by reducing their reservation trade. See also Ennis (2009) and Hu et al. (2014). We find these exercises insightful because they concern duration analysis, something for which search models are well suited while reduced-form models are, in a word, useless.

Rather than permanently distinct types, it is equivalent to have type determined each if the realization occurs before the CM closes – (27) still holds, with agents conditioning on all the information they have when they choose $\hat{a}$. Related to this, Berentsen et al. (2007) introduce banking. After the CM closes, but before the DM opens, agents realize if they will be buyers or not, at which point banks are open where they can take money out while others put it in. Or, to make it look more like standard deposit banking, we can say everyone puts money in the bank in the CM, and when types are revealed buyers withdraw while others leave their accounts alone. Thus,

[38] Comparing this to the planner's problem $\max_{q,n}\{\alpha(n)[u(q) - c(q)] - n\kappa\}$, it is easy to see that with competitive search $\iota = 0$ achieves efficiency on the intensive and extensive margins, $(q^*, n^*)$. In contrast, consider Kalai bargaining with DM entry by sellers. It is not hard to show this yields $n^*$ iff $1 - \theta = \alpha'(n^*)n^*/\alpha(n^*)$, the Hosios condition saying that sellers' share of the surplus should equal the elasticity of the matching function wrt their participation. Since competitive search yields $(q^*, n^*)$ automatically at $\iota = 0$, some say that it generates the Hosios condition endogenously.

a bank contract involves an option to convert interest-bearing deposits into cash (or to transfer the claim to a third party) on demand. This enhances efficiency because nonbuyers can earn interest on balances that would otherwise lay idle, with interest payments covered by what buyers pay for loans. Hence, banks reallocate liquidity towards those that could use more, similar to Diamond-Dybvig (1983), although here bankers realistically take deposits and make loans in cash rather than goods.

IN some applications, bankers can abscond with deposits or borrowers renege on debt (Berentsen et al. 2007). Others study the role of banks in investment and growth (Chiu and Meh 2011; Chiu et al. 2013). Related work includes Li (2006,2011), He et al. (2008), Becivenga and Camera (2011) and Li and Li (2013). In some of these, money and banking are complements, since a bank is where one goes to get cash; in others, they are substitutes, since currency and bank liabilities are alternative payment instruments, allowing one to discuss not only currency but checks or debit cards. Ferraris and Watanabe (2008,2011,2012) and He et al. (2013) have versions with investment in capital or housing used to secure cash loans from banks; as discussed below, in these models investment can be too high and endogenous dynamics can emerge. Williamson (2012,2013,2014) has models where banks design contracts like Diamond-Dybvig, incorporating multiple assets and private information. His banks hold diversified portfolios that allow depositors to share in the benefits of long-term investing with less exposure to liquidity shocks.[39]

Returning to participation, one can allow entry into the DM by sellers or buyers, or have a fixed population that choose to be one or the other (Rocheteau and Wright 2009; Faig 2008). This illustrates how search externalities might make $\iota > 0$ desirable. Aruoba et al. (2007) show that entry can be excessive, in which case $\iota > 0$ can raise welfare. This is important because it is a common misperception that $\iota = 0$ is always

[39] Gu et al. (2013*a*) develop a related theory of banking based on limited commitment. Agents have various combinations of attributes affecting the tightness of their IC. This determines who should be a banker, which means accepting and investing deposits. Banking can be essential in the same sense money can be (see also Araujo and Minetti 2011). Monitoring can also be endogenized, as in Diamond (1984). What is more novel is that agents use bank liabilities (claims on deposits) to facilitate transactions with third parties, as inside money. That this is perhaps the most commonly understood role of banking is clear from Selgin (2007): "Genuine banks are distinguished from other kinds of ... financial intermediaries by the readily transferable or 'spendable' nature of their IOUs, which allows those IOUs to serve as a means of exchange, that is, money. Commercial bank money today consists mainly of deposit balances that can be transferred either by means of paper orders known as checks or electronically using plastic 'debit' cards."

optimal in this class of models; that is false once there are trading frictions and externalities. And these are not trite examples, like saying that inflation is good because it taxes cash goods, like cigarettes, and someone has decided people smoke too much. That is trite because, without further assumptions, we can tax smoking directly. Relatedly, in New Keynesian models, deviations from the Friedman rule are due to nominal rigidities, but they can instead be addressed with fiscal policy (Correia et al. 2008). New Keynesian prescriptions of $\iota > 0$ follow from maintained inefficiencies, sticky prices, just as ours follow from congestion externalities, in both cases under the assumption that fiscal policy is not available.

We can add heterogenous DM meetings. Consider a random variable $\psi$, such that when a buyer meets a seller the former gets utility $\psi u(q)$ from the latter's goods. This illustrates further differences between the models here and ones with exogenous trading patterns, and how a strict lack of double coincidence meetings is not needed for money to be valued. Suppose in every meeting $i$ and $j$ like each other's output, but it may be asymmetric, $\psi_i \neq \psi_j$. With pure barter, where $i$ gets $q_i$ from $j$ and $j$ gets $q_j$ from $i$, it is easy to verify that efficiency entails $q_i > q_j$ whenever $\psi_i > \psi_j$. But bargaining yields the opposite, because $j$ drives a harder bargain when $i$ really likes his wares (to say it differently, $q_i$ is really expensive in terms of $q_j$ when $j$ does not like $i$'s goods very much). In a monetary economy, $i$ can pay in $d$ as well as $q$, and this improves his terms of trade. Here agents choose to use cash not because barter is impossible, but because it is coarse.[40]

We can also make the money supply random. Suppose $A = (1 + \pi) A_{-1}$, where $\pi$ is drawn at the start of the DM. The $\pi$ to be implemented later that period generally affects $\phi$, and hence $q$, but agents do not know it when they chose $\hat{a}$. Thus, agents equate the cost and expected benefit of liquidity. It is still optimal to have $\iota = 0$, but this is no longer equivalent to a unique money supply rule: a given $\iota$ is consistent with any stochastic process for $A$ with the same $E[1/(1 + \pi)]$. Lagos (2010$a$) characterizes the general class of monetary policies consistent with $\iota = 0$. If $d \leq a$ binds in every state, the results look like those in Wilson (1979) or Cole and Kocherlakota (1998)

[40] This approach was used in second-generation models by Engineer and Shi (1998,2001), Berentsen and Rocheteau (2002) and Jafarey and Masters (2003). Our presentation is based on Berentsen and Rocheteau (2003$a$). Relatedly, in Jacquet and Tan (2011,2012), money is more liquid than real assets with state-dependent dividends, since those are valued differently by agents with different hedging needs, while cash is valued uniformly.

for deterministic CIA models. See also Nosal and Rocheteau (2011), where buyers and sellers can be asymmetrically informed about inflation.

Another extension adds capital, although that can raise questions about competition between $k$ and $a$, addressed in Section 10. Here as in Aruoba and Wright (1993) we assume $k$ is not portable – cannot be taken to the DM – and claims to $k$ are not recognizable – they can be forged. So $k$ cannot be a medium of exchange. The CM production function is now $f(\ell, k)$, and the DM cost function $c(q, k)$. Then

$$\begin{aligned} W(a,k) &= \max_{x,\ell,\hat{a},\hat{k}} \left\{ U(x) - \ell + \beta V_{+1}(\hat{a},\hat{k}) \right\} \\ \text{st } x + \hat{k} &= \phi(a-\hat{a}) + w_\ell (1-t_\ell)\,\ell + \left[1 + (w_k - \delta_k)(1-t_k)\right] k + O, \end{aligned} \tag{30}$$

where $w_\ell$ and $w_k$ are factor prices, $t_\ell$ and $t_k$ are tax rates and $\delta_k$ is the depreciation rate. The FOC's imply $(\hat{a}, \hat{k})$ is independent of, and $W$ is linear in, $(a, k)$, generalizing our benchmark results. The setup also nests conventional macro: if $U(x) = \log(x)$ and $f(\ell, k)$ is Cobb-Douglas, nonmonetary equilibrium is exactly Hansen (1985).

As in Aruoba et al. (2011), at the start of the DM, assume preference/technology shocks determine which agents want to buy or sell. Then buyers search, while sellers sit on their $k$ and wait for buyers to show up, consistent with capital not being portable. The DM terms of trade satisfy $v(q, k) = \phi\hat{a}/w_\ell(1-t_\ell)$, where $v(q,k)$ comes from, say, Nash bargaining. It is not hard to derive the Euler equations for $a$ and $k$. Combining these with the other equilibrium conditions, we get

$$(1+\pi)\,v(q,k) = \beta c(q_{+1}, k_{+1})\left[1 + \alpha\sigma\lambda(q_{+1}, k_{+1})\right] \tag{31}$$

$$U'(x) = \beta U'(x_{+1})\left\{1 + \left[f_1(\ell_{+1}, k_{+1}) - \delta_k\right](1-t_k)\right\} - K(q_{+1}, k_{+1}) \tag{32}$$

$$U'(x) = 1/\left(1-t_\ell\right) f_1(\ell, k_t) \tag{33}$$

$$x + G = f(\ell, k) + (1-\delta_k)k - k_{+1}, \tag{34}$$

where $K(\cdot) \equiv \alpha\sigma\beta\left[c_2(\cdot) - c_1(\cdot)\,v_2(\cdot)/v_1(\cdot)\right]$. Given policy and $k_0$, equilibrium is a path for $(q, k, x, \ell)$ satisfying (31)-(34).

A novelty compared to standard macro is the term $K(\cdot)$ in (32), which reflects a holdup problem in the demand for capital, parallel to the one in the demand for money. The first problem is avoided iff $\theta = 0$ and the second iff $\theta = 1$; for intermediate $\theta$ there is underinvestment in both $\phi a$ and $k$, which has implications for welfare and the model's empirical performance. With bargaining, $\pi$ reduces investment by only a little. With price taking, the holdup problems vanish, and the calibrated model implies that eliminating a 10% inflation can increase steady state $k$ by 5%. This is sizable, although the welfare gain is lessened by the necessary transition. In any case, while we cannot go into more detail here, this should allay fears that these models are hard to integrate with mainstream macro. They are not.[41]

As regards labor markets, and monetary policy, the theory is flexible. If $c_k(q,k) = 0$ the model just presented dichotomizes, with $q$ solving (31), and $(k, x, \ell)$ solving (32)-(34). In this case $\partial q/\partial \pi < 0$, but $\pi$ does not affect $(k, x, \ell)$. The idea of putting $k$ in $c(q,k)$ was precisely to break the dichotomy, and leads to $\partial \ell/\partial \pi < 0$. Another way is to interact $q$ with $x$ in utility, say $U(q,x) - \ell$. Rocheteau et al. (2007) and Dong (2011) analyze the effect of $\pi$ on $\ell$ in models with indivisible labor, and hence unemployment. Since $\partial q/\partial \pi < 0$, when $U_{12} > 0$ we get $\partial x/\partial \pi < 0$, and given $\ell$ is used to make $x$ we get $\partial \ell/\partial \pi < 0$. When $U_{12} < 0$, however, $\partial \ell/\partial \pi > 0$. Hence, the Phillips curve slopes up or down depending on $U_{12}$. Moreover, this constitutes a fully-exploitable long-run tradeoff. Thus, even without nominal rigidities, the setup can be consistent with traditional Keynesian prescriptions: when $q$ and $x$ are substitutes, $U_{12} < 0$, permanently reducing unemployment by increasing $\pi$ is feasible. But it is not a good idea: the Friedman rule is still optimal.

Berentsen et al. (2011) take a different approach by adding a Pissarides (2000) frictional labor market, let's call it the LM and say it meets between the CM and DM, although the sequencing is not crucial.[42] Since labor is allocated in the LM, suppose CM utility linear in $x$, as in a typical Mortensen-Pissarides model. Then

$$W_e(a) = \max_{x,\hat{a}} \{x + \beta L_e(\hat{a})\} \text{ st } x = \phi(a - \hat{a}) + ew_1 + (1-e)w_0 + O,$$

where $L_e(a)$ is the LM value function indexed by employment status $e \in \{0, 1\}$, $w_1$ is employment income and $w_0$ is unemployment income. Let $w_1$ be determined by

[41] Aruoba's (2011) full dynamic-stochastic model performs empirically about as well as standard reduced-form models with flexible prices, matching many facts but not, e.g., the cyclicality or persistence of inflation. Venkateswarany and Wright (2013) is also a macro model with money, capital, taxes etc., but there money competes with other assets in the DM. Aruoba et al. (2014) have a version built to study housing, while Aruoba and Schorfheide (2011) have one with sticky prices. Also, $k$ is included in the model of Shi (1997$a$) in quantitative work by Shi (1999$a$,$b$), Shi and Wang (2006) and Menner (2006), and in the model of Molico (2006) by Molico and Zhang (2005).

[42] Shi (1998), Liu (2010), Lehmann (2012), Bethune et al. (2012), Gomis-Porqueras et al. (2013), Petrosky-Nadeau and Rocheteau (2013), Rocheteau and Rodriguez-Lopez (2013) and Dong and Xiao (2013) study related models. Independent of monetary applications (one can assume perfect credit), these models provide a neat way to put Mortensen-Pissarides into general equilibrium.

bargaining in the LM, but paid in the CM, without loss in generality. Thus $\hat{a}$ is independent of $a$, and also of $e$. Unemployed agents in the LM find jobs at a rate determined by a matching function taking unemployment and vacancies as inputs.

In the DM, worker-firm pairs produce output, but agents do not consume what they produce. Instead, all households and a measure $n$ of firms participate in the DM, where $n$ is the employment rate and hence also the measure of firms with workers. the DM arrival rate is determined by a matching function taking as inputs the measures of buyers and sellers. This establishes a connection between the LM and DM: higher $n$ allows buyers to trade more frequently, endogenizing the need for liquidity (something that is exogenous in many models). Thus, higher $n$ leads to higher $q$. A second link between is that higher $q$ leads to higher DM revenue for firms, which leads to more vacancies and higher employment. By taxing real balances, inflation lowers $q$, and this adversely affects employment. Smoothed unemployment rates are shown in Figure 6 from the data and a calibrated dynamic-stochastic version of Berentsen et al. (2011), generated by feeding in actual monetary policy, and assuming productivity, taxes etc. are constant. While it obviously cannot match every wiggle, and certainly misses the 60's, theory tracks broad movements in unemployment quite well.

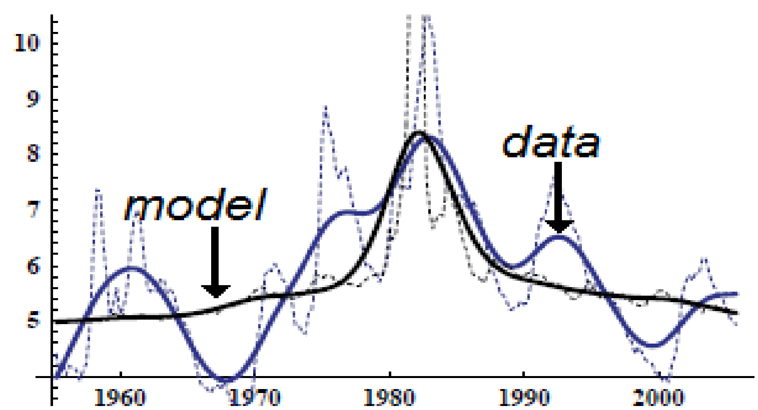


Figure 6: Unemployment: Model and Data

To be clear, despite lingering faith by some in a downward sloping Phillips curve, unemployment and inflation in US data are positively correlated, except maybe at very high frequencies or in subperiods like the 60's. In Figure 7, the left and right panels show unemployment against $\pi$ and $\iota$ for raw data and filtered data, color coded by decade (e.g., one can see the downward-sloping pattern of the 60's in pink). While the panels are similar, the one on the right is probably more relevant, since what really matters in the theory is expected inflation, and for that $\iota$ may better proxy than contemporaneous $\pi$. In any case, the relationship is positive in both panels. If one prefers more advanced time-series econometric methods, see Haug and King (2014), from which we display their remarkable Figure 8 on inflation and unemployment with a phase shift of 13 quarters. In terms of the Old Monetarist notion of "long and variable lags" between policy and outcomes, Figure 8 suggests they may be long, but they are not especially variable.[43]

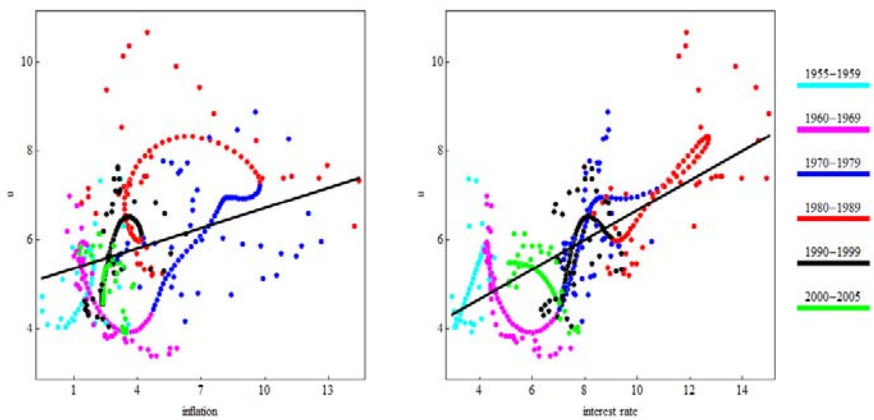


Figure 7: Scatters of Unemployment vs $\pi$ and $\iota$.

The framework can also explain nominal rigidities, as in Head et al. (2012), where the model matches well the micro evidence on price changes, but has implications very different from Keynesianism. The key ingredient is to replace bargaining in the DM with price posting à la Burdett and Judd (1983). This means each seller sets $p$ taking as given the distribution of other prices, say $G(p)$, and the behavior of buyers. The idea in Burdett-Judd is that the law of one price fails when some buyers see 1 and others see more than 1 price at a time. This delivers an endogenous $G(p)$ on

[43] Our findings are also related to Old Monetarism in other ways. Friedman (1968) says there may be a trade-off between inflation and unemployment in the short run, but in the longer run the latter gravitates to its "natural rate." Friedman (1977) says "This natural rate will tend to be attained when expectations are on average realized ... consistent with any absolute level of prices or of price change, provided allowance is made for the effect of price change on the real cost of holding money balances." This is the effect explicit in the above models. The data in Friedman (1977) suggested to him an upward-sloping Phillips curve, too, but certainly not as remarkablly as Figure 8.

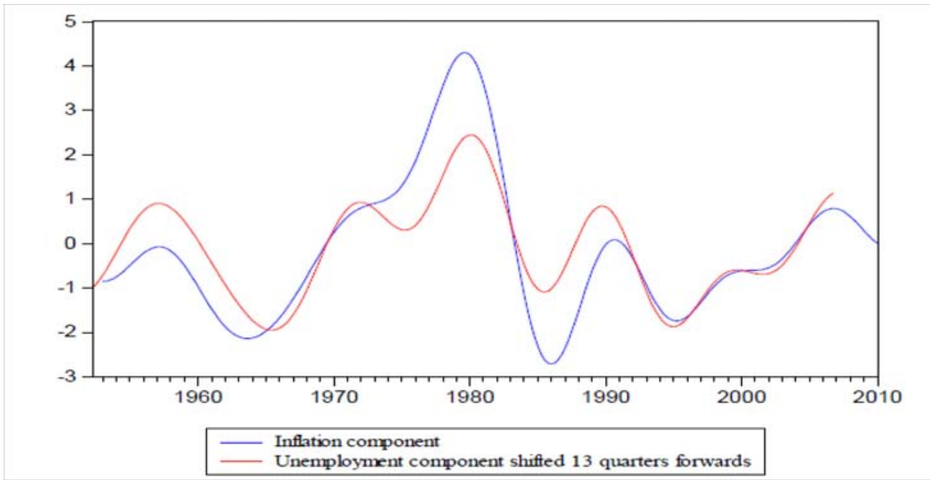


Figure 8: Shifted and Filtered Time Series, Unemployment and $\iota$.

an interval $\mathcal{P} = [\underline{p}, \overline{p}]$, and one can also easily solve for $G(p)$ explicitly. Profit is the same for all $p \in \mathcal{P}$, as high-$p$ sellers earn more per unit, while low-$p$ sellers earn less per unit but make it up on the volume.

The reason sticky nominal prices *seem* puzzling is this: Given inflation, if prices are sticky then sellers are letting their real price fall, when they ought to have a unique target price determined by real factors. The puzzle goes away once one understands that Burdett-Judd sellers do not have a unique target real price: all $p \in \mathcal{P}$ give the same profit. When the price level increases in stationary equilibrium, the distribution of nominal prices $G(p)$ shifts to keep the real distribution the same. But some individual sellers are content to keep their nominal price fixed because, again, they make it up in volume. Eventually they must change, since with inflation eventually $p \notin \mathcal{P}$, but individual prices can be constant for long durations.

Theory pins down $G(p)$ but not who sets which $p$. Consider a tie-breaking rule for sellers: if indifferent to changing $p$, change with some probability. By choosing this one free parameter, Head et al. (2010) find the implied price-change distribution fits the micro data in Klenow and Kryvtsov (2008) very well, as shown in Figure 9. It is also consistent with other facts hard to match parsimoniously with other models: an average price duration of just under a year; a large average price change yet many small changes; the probability and magnitude of adjustments are approximately independent of duration; inflation increases both the frequency and magnitude of

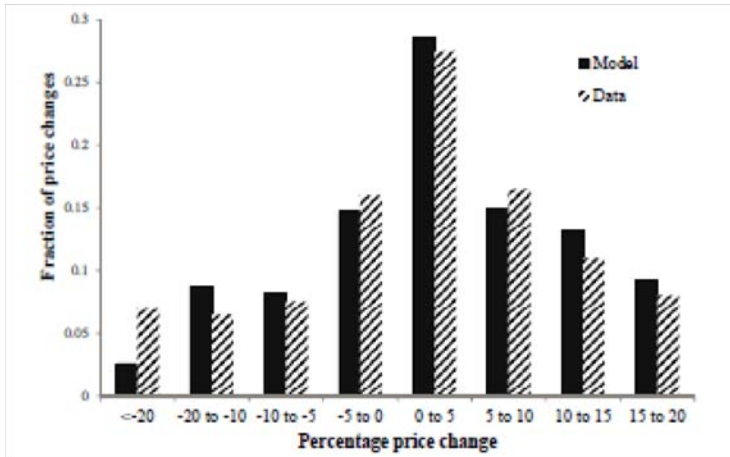


Figure 9: Price Change Distribution: Model and Data

changes; and inflation increases the positive fraction and reduces the negative fraction. But one cannot exploit this stickiness by policy. Different from models with ad hoc rigidities, a one-time increase in $A$ is met with a shift in $G(p)$ that leaves all real variables the same. Monetary policy is still relevant, since inflation matters, as in any New Monetarist model, but the effects are very different from Keynesian models.

See Wang (2011), Liu et al. (2014), Burdett et al. (2014) and Burdett and Menzio (2014) for more on sticky prices in this model. The latter is an important extension combining menu costs with the search frictions described above, both of which can generate nominal stickiness and price dispersion. Their quantitative work implies that search frictions are far and away more relevant – easily 70% of price dispersion in their data comes from that and not menu costs. This finding should shape the way we interpret observations on nominal prices: just because they appear sticky, one cannot conclude that menu costs are important nor that one should take seriously Keynesian policy recommendations. We again emphasize that explicitly modeling the exchange process, and in particular asking *why* prices are sticky, matters a lot.

One more way to make this point concerns studies of optimal monetary and fiscal policy by Aruoba and Chugh (2008) or Gomis-Porqueras and Peralta-Alva (2010), which overturn conventional wisdom from the reduced-form literature. MUF models, e.g., imply $\iota = 0$ is optimal even when other taxes are distortionary under the assumption that utility over goods and money is homothetic (Chari et al. 1991). But

Aruoba and Chugh's (2008) analysis shows that making utility homothetic over goods does *not* imply the value function is homothetic – again, microfoundations matter.[44]

# 9 The Cost of Inflation

Equilibrium allocations depend on $\iota$ or $\pi$, as well as the effective arrival rate $\alpha\sigma$ and mechanism $v(\cdot)$. This is important for policy analysis. Consider the typical quantitative exercise in reduced-form models, where one computes the cost of fully-anticipated inflation by asking how much consumption agents would be willing to give up to reduce $\pi$ from 10% to the level consistent with the Friedman rule. A consensus answer in the literature is around 1/2 of 1% (Cooley and Hansen 1989 and Lucas 2000 are well-known examples; see Aruoba et al. 2011 for more references). By contrast, in models along the lines of Lagos and Wright (2005), with Nash bargaining and $\theta$ calibrated to match retail markups, eliminating 10% inflation is worth around 5.0% of consumption – an order of magnitude higher than previously measured.

To explain this, following Craig and Rocheteau (2008*a*), consider Figure 10 that generalizes the traditional welfare-triangle analysis of Bailey (1956) and others. With Kalai bargaining, write (27) as

$$\iota(z) = \alpha\sigma \left\{ \frac{u'[q(z)]}{v'[q(z)]} - 1 \right\},$$

which can be interpreted as a money demand function with $z = \phi a$ denoting real balances. As $\iota \to 0$, $z \to z_0 = \theta c(q^*) + (1-\theta)u(q^*)$, and there is an upper bound $\bar{\iota} = \alpha\sigma\theta/(1-\theta)$ above which $z = 0$. If $\theta = 1$, then $z_0 = c(q^*)$ and $\bar{\iota} = \infty$, and in this case the welfare cost of going from $\iota = 0$ to $\iota_1 > 0$ is captured by the area under the curve, $ABC$, because

$$\int_{z_1}^{z_0} \iota(z)dz = \alpha\sigma \{u[q(z_0)] - c[q(z_0)]\} - \alpha\sigma \{u[q(z_1)] - c[q(z_1)]\}.$$

[44] Other applications include Boel and Camera (2006), Camera and Li (2008), Li (2001,2007), Sanches and Williamson (2010) and Berentsen and Waller (2011*a*), who study interactions between money and bonds/credit. Berentsen and Monnet (2008), Berentsen and Waller (2011*b*) and Andolfatto (2010,2013) discuss policy implementation. Guerrieri and Lorenzoni (2009) discuss liquidity and cycles. Silveira and Wright (2009), Chiu and Meh (2011) and Chiu et al. (2013) study the the role of liquidity and intermediation in the market for ideas and in endogenous growth models. Other growth applications are Waller (2011) and Berentsen et al. (2012). Duffy and Puzzello (2013) experiment in the lab with a finite population, where good outcomes can be supported by social norms, without money. Yet their subjects tend to favor money. They interpret this as saying money is a coordination device.

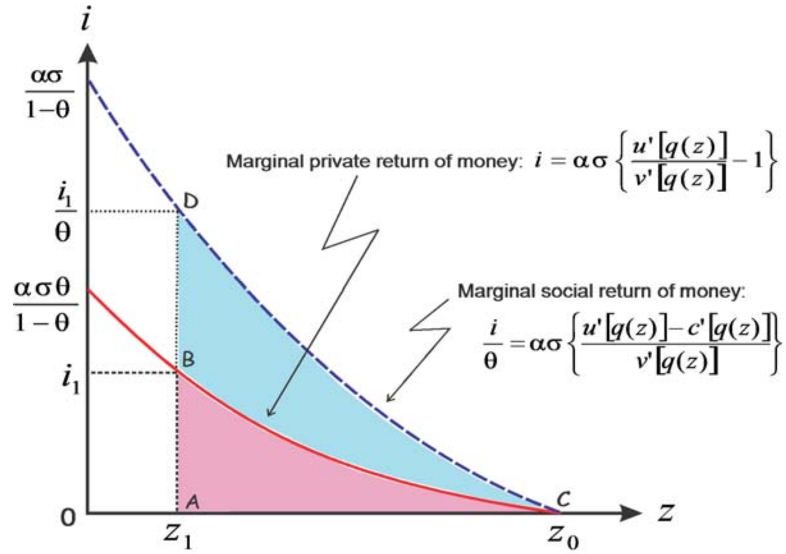


Figure 10: Welfare Triangles and the Cost of Inflation

But if $\theta < 1$, the cost of inflation no longer coincides with this area, as buyers receive only a fraction of the increase in surplus coming from $z$. The true cost of inflation is the area $ADC$, and not $ABC$, because $ABC$ ignores the surplus of the seller.[45]

To quantify the effects, a typical strategy is this: Take $U(x) = \log(x)$, $u(q) = \Gamma q^{1-\gamma}/(1-\gamma)$ and $c(q) = x$. Then set $\beta$ to match an average real interest rate. Then, since it is not so easy to identify, simply set $\alpha\sigma = 1/2$ and later check robustness (it does not matter much over a reasonable range). Then set $\theta$ to match a markup of 30%.[46] Finally, set the DM utility parameters $\Gamma$ and $\gamma$ to match money demand in the data. A typical fit is shown in Figure 11, which is drawn using $M1$ data of the US, although there are alternatives (it is probably better to use the $M1S$ data discussed in Cynamon et al. 2006, or better yet, the data in Lucas and Nicolini 2013, but the goal here is mainly to illustrate the method). This delivers results close to Lagos and

[45] With Nash bargaining, the horizontal intercept $z_0$ is below the $z$ that gives $q^*$. Based on this, Lagos and Wright (2005) explained the results as follows: competitive pricing delivers the first best at $\iota = 0$ and so, by the envelope theorem, a small $\iota > 0$ has only a second-order effect; but with Nash bargaining and $\theta < 1$, $\iota = 0$ does not deliver the first best, so the envelope theorem does not apply. However, Kalai bargaining does deliver $q^*$ at $\iota = 0$, even with $\theta < 1$, yet the cost of 10% inflation is similar with Kalai and Nash bargaining. An interpretation consistent with this is that bargaining implies a holdup problem that is magnified as $\iota$ increases.

[46] Lagos and Wright (2005) targeted a markup of 10%, but the results are similar, since 10% already makes $\theta$ small enough to matter. Our preferred 30% is based on the Annual Retail Trade Survey (see Faig and Jerez 2005).

Wright (2005). Interestingly, these calibrations do not imply the DM accounts for a large fraction of total output (it accounts for about 10% of real output).[47]

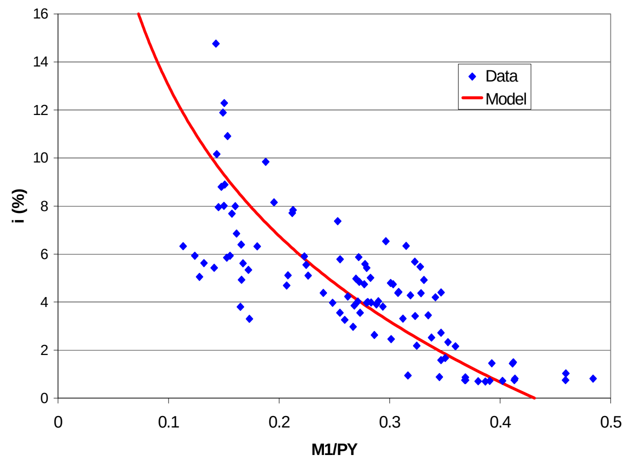


Figure 11: Money Demand: Theory and Data

In an extension,with private information, Ennis (2008) finds even larger effects, between 6% and 7%. Dong (2010) allows inflation to affect the variety of goods, and hence exchange on the extensive and intensive margins. She gets numbers between 5% and 8% with bargaining. Wang (2011) uses Burdett-Judd pricing to get price dispersion, so that inflation affects both the average price and dispersion. He gets 3% in a calibration trying to match price dispersion, and 7% when he matches a 30% markup. In a model with capital, Aruoba et al. (2011) get around 3% across steady states, although some of that is lost during transitions. Faig and Li (2009) add a signal extraction problem and decompose the cost into components due to anticipated and unanticipated inflation; they find the former is far more important. Aruoba et al. (2014) add home production, which increases the cost of inflation somewhat, if not dramatically. Dutu (2006) and Boel and Camera (2011) consider other countries. Boel and Camera (2009) consider heterogeneity.

Rocheteau and Wright (2009), Faig and Jerez (2006) and Dong (2010) use competitive search, and find costs closer to 1% or 1.5%, because this mechanism avoids holdup problems. Relatedly, in the model with capital in Aruoba et al. (2011),

[47] Also, as one changes the model frequency from annual to quarterly or monthly, the relevant value of $\alpha\sigma$ changes, keeping the substantive results the same – a big advantage over CIA models where agents spend all their money every period.

switching from bargaining to Walrasian pricing similarly brings the cost down, even though the effect of $\pi$ on investment is much bigger than with bargaining. Although the choice of mechanisms clearly matters for the results, models with bargaining, price taking and posting can all match the money demand data about as well. Rocheteau (2012) also shows that a socially efficient mechanism like the one in Hu et al. (2009) can match money demand, and it implies the welfare cost of 10% inflation is 0. These results underscore the importance of better understanding the microfoundations of price formation in decentralized markets.

A more radical extension is Aruoba and Schorfheide (2011), who estimate a model integrating New Monetarist and Keynesian features. They compare the importance of sticky price distortions, which imply $\pi = 0$ is optimal, and the effect emphasized here, which implies $\iota = 0$. They estimate the model under four scenarios, depending on the DM mechanism and whether they fit short- or long-run money demand. With bargaining and short-run demand, despite large sticky-price distortions, $\iota = 0$ is optimal. The other scenarios even with parameter uncertainty never imply $\pi = 0$. Craig and Rocheteau (2008*b*) reach similar conclusions in a menu-cost version of our benchmark model, as in Benabou (1988,1992) and Diamond (1993) in setups where money is merely a unit of account. It matters: Diamond (1993) argues inflation usefully erodes the market power of sellers; but Craig and Rocheteau show that that is dwarfed by the inefficiency emphasized here for any reasonable parameters. Deflation, not inflation, is optimal.

Sometimes some inflation can be good. In Craig and Rocheteau (2008*b*) or Rocheteau and Wright (2009), with entry or search-intensity, optimal $\pi$ is around 2%. In Venkateswarany and Wright (2013), capital taxation makes $k$ too low, and since inflation partially offsets that, optimal $\pi$ is around 3.5% (obviously a second-best result). In a model with nondegenerate $F(a)$, Molico (2006) illustrates numerically the possibility of a positive redistributive effect. In Chiu and Molico's (2010,2011) calibrated models, this effect reduces the cost of inflation, but optimal $\pi$ is negative. The same is true of Dressler (2011*a*,*b*), although his majority-voting equilibrium has $\iota > 0$. While understanding the effects of inflation is still work in progress, based on all this, we see no evidence for the dogmatic position of many Keynesians that it is fine to ignore monetary considerations of the type considered here.

# 10 Liquidity and Finance

Assets other than currency also convey liquidity.[48] To begin this discussion, we emphasize how assets can facilitate transactions in different ways. First, it seems clear that with perfect credit, as discussed below, there is no such role for assets. But if sellers worry you will renege, they may insist on getting something, like an asset, by way of quid pro quo. Or, instead, as in Kiyotaki and Moore (1997), they may require assets to serve as *collateral* that can be seized to punish those who default.

To pursue this, first consider our benchmark model with perfect credit. Then

$$W(a, D) = \max \{U(x) - \ell + \beta V(\hat{a})\} \text{ st } x = \phi(a - \hat{a}) + \rho a + \ell + O - D,$$

where $D$ is debt coming from the last DM, denominated in $x$, which is now a settlement instrument, and one-period debt is imposed without loss of generality. Perfect credit means default is not an option, so promises are constrained only by total resources, which we assume is not binding. Thus any $q$ is available in the DM if a buyer promises to make a payment in the CM of $D = v(q)$. For reasonable mechanisms, this implies $q = q^*$ and $V(a) = W(a, 0) + \alpha\sigma [u(q^*) - c(q^*)]$. Since $a$ does not affect this, the Euler equation is $\phi_{-1} = \beta(\rho + \phi)$, the only bounded solution to which is the constant "fundamental" price $\phi = \phi^* \equiv \rho/r$.

One can add exogenous default (e.g., debtors may die) by multiplying the LHS of $D = v(q)$ by the repayment probability; nothing else changes. For strategic default, suppose to make a point that we cannot take away defaulters' future credit – perhaps they can move to new towns, where they are anonymous, or, in the notation of Section 3, $\mu = 0$. Then the only punishment for default is taking away assets that have been pledged as security, assuming for simplicity they have been assigned to third parties with commitment. Let us also assume that one can pledge a fraction $\chi \leq 1$ of one's

[48] Papers where real assets facilitate trade in our benchmark model include Geromichalos et al. (2007), who have equity $a$ in fixed supply, and Lagos and Rocheteau (2008*a*), who have reproducible capital $k$. There assets have properties (portability, recognizability etc.) making them, like cash, usable in DM trade. See also Rocheteau (2011), Lester et al. (2002), Li et al. (2012), Rocheteau and Petrosky-Nadeau (2012), Nosal and Rocheteau (2013), Hu and Rocheteau (2013) and Venkateswarany and Wright (2013). There are applications using the model to study financial issues like the credit-card-debt puzzle (Telyukova and Wright 2008), on-the-run phenomena (Vayanos and Weill 2008), the equity-premium and risk-free-rate puzzles (Lagos 2010*b*,2011), home bias (Geromichalos and Simonovska 2014), repos (Narajabad and Monnet 2012), the term structure (Geromichalos et al. 2013; Williamson 2013) and housing bubbles (He et al. 2013).

assets (Section 11 discusses endogenizing this). If you owe $D$ secured by $\chi a$, the IC for honoring the obligation is $D \leq (\phi + \rho) \chi a$, since for any greater $D$ you would rather forfeit the collateral. Hence, the debt limit is $\bar{D} = (\phi + \rho) \chi a$. One can also add an intercept, so that $\bar{D} = D_0 + (\phi + \rho) \chi a$, where $D_0$ represents an unsecured credit limit, perhaps supported by other punishments.

Notice something: rather than promising $\bar{D} = (\phi + \rho) \chi a$, you may as well turn over $\chi a$ assets and finalize the transaction in the DM. In other words, it is equivalent for assets to serve as a medium of exchange or as collateral, if you have the assets to turn over, and it you don't have them it is not clear how they can serve as collateral.[49] One can imagine exceptions – if it is "inconvenient" to turn over part of your house in payment, you prefer to use a home-equity loan – but unless that is modeled explicitly, which must mean more than evoking "convenience," secured credit à la Kiyotaki-Moore is not distinct from assets serving as money à la Kiyotaki-Wright.

An ostensible distinction is that with secured credit only a fraction $\chi$ of your assets can be pledged, but, formally, we can just as well say you can only hand over a fraction $\chi$ of your assets, and while one might tell different stories, the equations are the same. Another ostensible distinction is that the Kiyotaki-Moore literature usually talks in terms of producer credit while here we talk mainly about consumer credit, but, in terms of theory, as we said earlier that is a relabeling. A less delusory distinction may be this: In the models presented above, assets are useful for the acquisition of $q$. Now suppose what you want is not $q$, but more of the same asset, like a producer increasing business capital or a homeowner increasing housing capital. It will not do to exchange old $k$ for new $k$. But you might manage by pledging old $k$ to get new $k$ on credit *if* the former is pledgeable as collateral while the latter is not, which is arbitrary but perhaps not crazy.

In any case, we now include in the model the Kiyotaki-Moore parameter $\chi$, and consider assets with $\rho > 0$. Now we cannot be sure $d \leq \chi (\phi + \rho) a$ binds, the way it must with fiat money. If we provisionally assume it binds, the analog of (25) is

$$\phi_{-1} = \beta (\phi + \rho) [1 + \alpha \sigma \chi \lambda (q)] , \tag{35}$$

[49] From David Andolfatto's blog: "On the surface, these two methods of payment look rather different. The first entails immediate settlement, while the second entails delayed settlement. To the extent that the asset in question circulates widely as a device used for immediate settlement, it is called money ... To the extent it is used in support of debt, it is called collateral. But while the monetary and credit transactions just described look different on the surface, they are equivalent."

where $\lambda(q)$ is as before. Using $v(q) = \chi(\phi + \rho)a$ to eliminate $\phi$ and $\phi_{-1}$, we get a difference equation in $q$ analogous to (26). While $\rho > 0$ rules out equilibria where $\phi = 0$ or $\phi \to 0$, there can still exist cyclic, chaotic and stochastic equilibria (see Zhou 2003, Rocheteau and Wright 2013 and references therein). Importantly, it is not the fiat nature of money that leads to interesting dynamics, but an inherent feature of liquidity, which applies whether assets serve as a means of payment or collateral, and whether they are nominal or real.

Notice that while $\chi$ usually does not appear in Kiyotaki-Wright models, or $\alpha\sigma$ in Kiyotaki-Moore models, it looks from (35) that only the product $\alpha\sigma\chi$ matters. That is not quite right, though, because $v(q) = \chi(\phi + \rho)a$, and this together with (35) determine $q$ and $\phi$ jointly. One can check that $\phi$ is increasing in $\alpha\sigma$ and nonmonotone in $\chi$, so they affect the system differently.[50] Moreover, we claim the nonmonotonicity of $\phi$ in $\chi$ is inescapable, for the following reason. We know that $\phi = \phi^*$ in two cases: (i) when $\chi = 0$; and (ii) when $\chi$ is so big the liquidity constraint is slack (assuming liquid assets are not too scarce). Since we can have $\phi > \phi^*$ for intermediate values of $\chi$, the claim follows, just like the Laffer curve (similarly, in Lagos 2010*b* the liquidity premium is nonmonotonic in $\theta$). One implication is that as assets become more pledgeable, their prices first rises then fall. This can generate a housing boom and bust as home-equity lending becomes more liberal (He et al. 2013).

This is all predicated on the liquidity constraint binding, which means $\lambda(q) > 0$. We now check this. In stationary equilibrium, using $v(q) = \chi(\phi + \rho)a$ to eliminate $q$ from (35), we can interpret the result as the long-run demand for $a$ as a function of $\phi$. One can check demand is decreasing (as in Wright 2010) for $q < q_0$, defined by $\lambda(q_0) = 0$. Define $A_0$ by $v(q_0) = \chi(\phi^* + \rho)A_0$, where $\phi^* = \rho/r$ is the "fundamental" price of the asset. Then $A_0$ is the asset position at which an agent satiates in liquidity. Hence, the asset demand curve is as shown in Figure 12, derived by truncating (35) at $\phi^*$. The following is now immediate: if the fixed supply is $A \geq A_0$ then liquidity is plentiful and $\phi = \phi^*$; but if $A < A_0$ then liquidity is scarce and $a$ bears a premium $\phi > \phi^*$. See Nosal and Rocheteau (2011) and Kiyotaki and Moore (2005) for more discussion.

[50] If we use one-period instead of infinite-lived assets, the equilibrium conditions are $\phi_{-1} = \beta\rho[1 + \alpha\sigma\chi\lambda(q)]$ and $v(q) = \rho\chi a$. We mention this because with one-period assets there is another distinction between $\chi$ and $\alpha\sigma$: $\partial q/\partial\chi > 0$ and $\partial q/\partial\alpha\sigma = 0$.

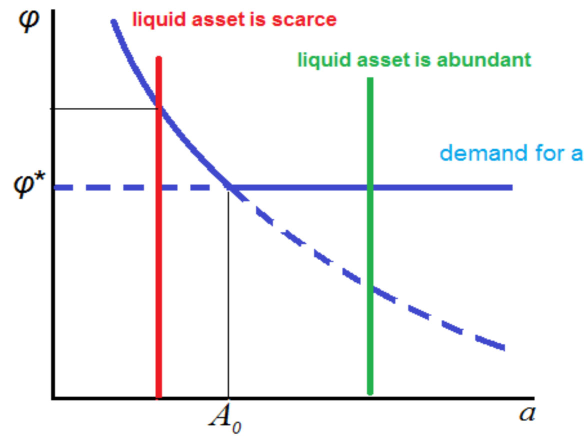


Figure 12: Asset Supply and Demand

The above analysis applies when asset supply is fixed, as in Geromichalos et al. (2007) or Lagos (2010*a*,2010*b*,2011). The analysis of neoclassical capital, as in Lagos and Rocheteau (2008*a*), is similar except the supply curve is horizontal instead of vertical, since the price of $k$ in terms of $x$ is 1 when they are the same physical object. In this case, liquidity considerations are manifest not by $\phi > \phi^*$ but by $k > k^*$. Housing as in He et al. (2013) is similar, when it conveys liquidity by securing home-equity loans, except supply need not be horizontal or vertical, so both price and quantity can be affected by liquidity considerations. Recent research considers models with different combinations of these assets and currency.

Now, to discuss OMO's (open market operations), suppose there is fiat money plus a real, one-period, pure-discount, government bond that is issued in one CM and pays a unit of numeraire in the next CM.[51] Here bonds are *partially liquid* – i.e., acceptable in *some* DM meetings. The government budget constraint is $\pi\phi_m A_m = T + G + A_b(1 - \phi_b)$, assuming $A_b$ is constant, to generate stationarity outcomes, and $T$ adjusts to balance the budget (recall $T$ affects $\ell$ but nothing else). As usual, $A_m$ grows at rate $\pi$, and in stationary equilibrium $\phi_{m,-1}/\phi_m = 1 + \pi$. The nominal interest rate on an *illiquid* real and nominal bonds are still $1 + r = 1/\beta$ and $1 + \iota = (1 + \pi)/\beta$. The nominal return on the liquid bond (the amount of cash one can get in the next

[51] This discussion is based on Rocheteau et al. (2014), which is a simplification of Williamson (2012,2013,2014) and Rocheteau and Rodriguez (2013). As in those papers one can easily study nominal bonds, too.

CM by putting a dollar into real bonds in this CM) is $\iota_b$, and generally differs from $\iota$. A straightforward application of the techniques used above lead to the FOC's $\phi_{b,-1} = \beta V_1(\hat{a}_m, \hat{a}_b)$ and $\phi_{b,-1} = \beta V_2(\hat{a}_m, \hat{a}_b)$.

In the DM, with probability $\alpha_m$ a buyer meets a seller that accepts only $a_m$, whence he gets $q_m$; with probability $\alpha_b$ a buyer meets one that accepts only $a_b$, whence he gets $q_b$; and with probability $\alpha_2$ a buyer meets one that accepts both, whence he gets $q_2$. A realistic special case is $\alpha_b = 0$. Special cases of are ones where the assets are perfect substitutes, $\alpha_b = \alpha_m = 0$, and where only cash is liquid, $\alpha_b = \alpha_2 = 0$. In any case, we know that buyers must spend all their money in a type-$m$ meeting, and may as well spend it all in a type-2 meeting since then both parties are indifferent between combinations of $a_m$ and $a_b$ with the same value. Now, we know $d_2 \leq d_b$, in general, but there are still three possible cases: (i) $d_b = d_2 = a_b$; (ii) $d_b = a_b > d_2$; and (iii) $a_b > d_b > d_2$.

Consider case (i), and assume monetary equilibrium exists, which is valid at least if $\iota$ is not too high. After some algebra, the Euler equations can be written

$$\iota = \alpha_m \lambda(q_m) + \alpha_2 \lambda(q_2) \tag{36}$$

$$s = \alpha_b \lambda(q_b) + \alpha_2 \lambda(q_2), \tag{37}$$

where $s \equiv (\iota - \iota_b) / (1 + \iota_b)$ is the spread between the returns on illiquid and liquid bonds (as used in Wright 2010, Silveira and Wright 2010, and Rocheteau and Rodriguez 2013). Just like $\iota$ is the return differential between illiquid bonds and currency, $s$ is the differential between illiquid and liquid bonds, and both measure the cost of liquidity. It is also reminiscent of the "convenience yield" in Krishnamurthy and Vissing-Jorgensen (2012) that they measure by the difference between yields on government and corporate bonds.[52]

Notice standard accounting yields

$$\iota_b = \frac{\alpha_m \lambda(q_m) - \alpha_b \lambda(q_b)}{1 + \alpha_b \lambda(q_b) + \alpha_2 \lambda(q_2)}. \tag{38}$$

This implies $\iota_b \gtreqless 0$ iff $\alpha_m \lambda(q_m) \gtreqless \alpha_b \lambda(q_b)$. Hence, liquid bonds can have a *negative* nominal return, $\iota_b < 0$, as is sometimes seen in reality (*Wall Street Journal*, Aug. 10,

[52] One may be tempted to say our model rationalizes their reduced-form specification with T-bills in the utility function, although it's not clear if that is desirable, any more than saying the models presented above rationalize money in the utility function. The sine qua non of our approach is modeling exchange explicitly, not deriving indirect utility functions from deeper primitives.

2012), even though we cannot have $\iota < 0$ for illiquid bonds. When $\iota_b < 0$, individuals hold $a_b$ because it's expected liquidity premium exceeds that on currency. Thus, $\iota_b < 0$ is impossible when $\alpha_b = 0$ or $q_b = q^*$, and in particular $\alpha_b = \alpha_2 = 0$ implies $\iota_b = \iota$. Importantly, $\iota_b < 0$ does not violate standard no-arbitrage conditions because individuals cannot hold $a_b < 0$ – i.e., while anyone can issue a bond, they cannot guarantee that it will circulate in the DM.[53]

Returning to the characterization of equilibrium in case (i), $(q_m, q_b, q_2, z, s)$ solves $v(q_m) = z$, $v(q_b) = A_b$, $v(q_2) = z + A_b$, (36) and (37), while $(x, \ell)$ solves the usual CM conditions. It is routine to derive the effects of inflation or the nominal interest rate on illiquid bonds, and in particular $\partial z/\partial \iota < 0$, $\partial q_m/\partial \iota < 0$, $\partial q_b/\partial \iota = 0$ and $\partial q_2/\partial \iota < 0$. Also, as long as $\alpha_m > 0$ or $\alpha_2 > 0$, which means bonds are sometimes accepted in the DM, higher $\iota$ raises the spread $s$, or equivalently decreases $\iota_b$ and increases $\phi_b$, as agents try to shift from $a_m$ to $a_b$. As regards an OMO, say selling bonds for cash, since a one-time change in $A_m$ is as always neutral, this is the same as simply increasing $A_b$. Then $\partial z/\partial A_b < 0$ and $\partial q_m/\partial A_b < 0$ as long as $\alpha_2 > 0$, since higher $A_b$ makes liquidity less scarce in type-2 meetings, so agents economize on $z$, which hurts in type-$m$ meetings. Similarly, $\partial q_b/\partial A_b > 0$, and $\partial q_2/A_b > 0$ as long as $\alpha_m > 0$. If $\alpha_2 > 0$ and $\alpha_m > 0$, by makes liquidity less scarce, higher $A_b$ reduces $s$ and $\phi_b$ while increasing $\iota_b$. Basically, an OMO reallocates liquidity between situations where bonds are accepted and where they are not.[54]

Consider now a modification where agents learn what type of meeting they will have in the DM before leaving the CM. Then without loss of generality, buyers who will have a type-$m$ meeting carry only cash to the DM while those who will have a type-2 meeting carry bonds and possibly cash. If $A_b$ is big then $q_b = q_2 = q^*$, no one takes cash to meetings where bonds are accepted, and an OMO has no effect at the margin. If $A_b$ is lower, we get $\iota_b \in (0, \iota)$ and a change in $A_b$ affects $\iota_b$ as well as the allocation. If $A_b$ is too small then agents take some cash to type-2 meetings, and $q_m = q_2 < q^*$ despite $\iota_b = 0$. This last outcome looks like a "liquidity trap" where

[53] He et al. (2008) or Sanches and Williamson (2010) also have liquid assets with negative nominal returns if they are less susceptible to loss or theft than cash.

[54] These results are for case (i). In cases (ii) and (iii) $z$ is determined as in a pure-currency economy. The case that obtains depends on policy: if bonds are abundant, we get case (iii); if they are less abundant, we get case (i) when $\iota$ is high and (ii) when $\iota$ is low. If $\alpha_2 > 0 = \alpha_m = \alpha_b$, so bonds and money are perfect substitutes, an OMO is irrelevant because changes in $A_b$ are offset by endogenous changes in $z$, as in Wallace (1981,1983).

output is inefficiently low, the nominal rate on liquid bonds is 0, and OMO's are ineffectual. Again, these models are useful in trying to understand policy. And while the preceding discussion concerned conventional monetary policy, swapping T-bills for cash, one can obviously extend it to unconventional policy.

At this juncture we offer some general comments on the coexistence of money and riskless, perfectly recognizable, nominal bonds with positive returns. Papers trying to address this coexistence (or rate-of-return dominance) puzzle generally impose some asymmetry between money and nominal bonds. An example is Aiyagari et al. (1996), a second-generation model with money and two-period government bonds. As in Li (1994,1995), Li and Wright (1998) or Aiyagari and Wallace (1997), there are government agents that act like anyone else except: in meetings with private sellers, they may either pay with money or issue a bond; in meetings with private buyers, they may refuse to accept not-yet-matured bonds; and in any meeting they can turn matured bonds into cash. Equilibria with valued money and interest-bearing bonds exist because of asymmetry in the way government treats the assets. See also Zhu and Wallace (2007), where the trading protocol gives a larger share of the surplus to agents with more money.

Such asymmetries are adopted because of a belief that, under laissez faire, absent exogenous assumptions that favor money there are no equilibria where it coexists with default-free, interest-paying, nominal bearer bonds. Yet arguably there are historical instances where such securities and money both circulated – a strong instance of the rate-of-return-dominance puzzle. Lagos (2013$a$) addresses this in a version of Lagos-Wright (2005) where currency consists of notes that are heterogeneous in extraneous attributes called "moneyspots" to make a connection with sunspots; these can be any payoff-irrelevant characteristic, like serial numbers. He shows this is all one needs to get equilibria where money coexists with interest-bearing bonds. Heuristically, in equilibrium the extraneous attributes are priced, so that different notes are valued differently, supported purely by beliefs, and this allows money and nominal bonds to coexist (see Lagos 2013$a$,$b$ and Wallace 2013 for more discussion).

Moving from pure theory to more applied issue, Lagos (2008,2010$b$) shows how liquidity helps us understand two of the best-known issues in finance, the risk-free-rate and the equity-premium puzzles. There are two real assets, a one-period risk-free government bond, and shares in a "tree" with random dividend $\rho$, with prices $\phi_b$ and $\phi_s$. In a minor modification of the benchmark model, returns are in terms of a second CM consumption good $y$, which is numeraire, while $p_x$ is the price of $x$, and CM utility is $U(x,y)$. Here both assets can be used in all DM transactions, $\alpha_2 > 0 = \alpha_b = \alpha_s$, but we consider $\alpha_b, \alpha_s > 0$ below. The Euler equations are

$$U_2\left[x\left(\rho_{-1}\right), \rho_{-1}\right]\phi_{b,-1} = \beta\mathbb{E}\left\{U_2\left[x(\rho),\rho\right]\left[1+\alpha_2\lambda(q)\right]\right\} \tag{39}$$

$$U_2\left[x\left(\rho_{-1}\right), \rho_{-1}\right]\phi_{s,-1} = \beta\mathbb{E}\left\{U_2\left[x(\rho),\rho\right](\phi_s+\rho)\left[1+\alpha_2\lambda(q)\right]\right\} \tag{40}$$

using feasibility, $y=\rho$, where $x(y)$ solves $U_1(x,y)=1$.

Compared to our benchmark, now expected marginal utility $U_2(\cdot)$ at different dates matters; compared to asset-pricing models following Mehra and Prescott (1985), now liquidity matters. From (39)-(40) follow a pair of restrictions,

$$\mathbb{E}(\Omega R_b - 1) = \xi_b \tag{41}$$

$$\mathbb{E}[\Omega(R_s - R_b)] = \xi_s, \tag{42}$$

between the MRS, $\Omega \equiv \beta U_2(x,\rho)/U_2\left(x_{-1},\rho_{-1}\right)$, and measured returns, $R_b \equiv 1/\phi_{b,-1}$ and $R_s \equiv (\phi_s+\rho)/\phi_{s,-1}$. For Mehra-Prescott, $\xi_b = \xi_s = 0$, and for standard preferences this is violated by data, where $\xi_b < 0$ and $\xi_s > 0$. By contrast, here

$$\xi_b = -\alpha_2\mathbb{E}\left[\Omega\lambda(q)R_b\right] \tag{43}$$

$$\xi_s = \alpha_2\mathbb{E}\left[\Omega\lambda(q)(R_b - R_s)\right]. \tag{44}$$

From (43), $\xi_b < 0$ as long as $\alpha_2\lambda(q_{t+1}) > 0$, which says the bond has liquidity value in some state of the world. Hence, a liquidity-based model is always at least qualitatively consistent with the risk-free-rate puzzle. Also, $\xi_s$ is the weighted average return differential $R_b - R_s$ across states. This is generally ambiguous in sign, but suppose there is a high- and a low-growth state, and $R_b - R_s$ is positive in the latter and negative in the former. If the weight $\Omega\lambda(q)$ tends to be larger in the low state, then $\xi_s > 0$. Hence, even without giving bonds an liquidity advantage over equity, the model rationalizes the equity-premium puzzle. In a calibration with $U(x,y) = \hat{U}(x) + y^{1-\gamma}/(1-\gamma)$, the model is quantitatively consistent with both puzzles for $\gamma = 10$, while standard calibrations of Mehra-Prescott need $\gamma = 20$.

Modest differences in acceptability matter a lot. If in 2% of DM trades buyers can only use bonds, while in the remaining 98% they can use bonds or shares, then $\gamma = 4$

is enough to generate an equity premium of 7% – 10 times larger than Mehra-Prescott with $\gamma = 4$. The broader point is that these models have a lot of potential for financial economics, and can have further implications for policy. In Lagos (2010$a$,2011), currency and claims to a random aggregate endowment can both be used in the DM, and in some states asset values are too low to get $q^*$. In such a model, monetary policy can mitigate this by offsetting liquidity shortages. Even more generally, the message is that liquidity considerations have important implications for the effects of monetary policy on asset markets. This is no surprise. What we are suggesting is a tractable GE framework that can be used to make this precise.[55]

# 11 Information and Liquidity

Going back to Law (1705), Jevons (1875) and others, one approach to understanding the moneyness of assets appeals to informational frictions. Alchain (1977) iconoclastically goes so far as to say "It is not the absence of a double coincidence of wants, nor of the costs of searching out the market of potential buyers and sellers of various goods, nor of record keeping, but the costliness of information about the attributes of goods available for exchange that induces the use of money." While all of the models discussed above need some information frictions to make money essential, here we have in mind private information about the quality of goods or assets.[56]

Williamson and Wright (1994) has no double coincidence problem, but barter is hindered by agents trying to trade goods that are "lemons" – they are cheaper to

[55] In related work, Lagos (2010a) shows that a large class of money growth rules, as functions of the state, implement $\iota = 0$ and make asset prices independent of monetary considerations. But one can alternatively target a constant $\iota > 0$, implying they depend on policy. In Lagos (2011), deviations from an optimal policy make asset prices persistently deviate from "fundamental" values. To support a constant $\iota$ the growth rate of the money supply must be low in states where real asset values are low, introducing a negative relationship between $\iota$ and returns, with implications for macro aggregates generally. Even if variations in output are exogenous, a positive correlation between inflation and output emerges from targeting constant $\iota$.

[56] Recent studies of adverse selection in decentralized asset markets include Guerrieri et al. (2010), Chiu and Koeppl (2011), Chang (2012) and Camargo and Lester (2013), but there assets play no role in facilitating exchange. First-generation models with information frictions and assets playing this role include Cuadras-Morató (1994), Williamson and Wright (1994), Li (1995,1998), Kim (1996), Okumura (2006), and Lester et al. (2011). Second-generation models include Trejos (1997,1999), Velde et al. (1999), Burdett et al. (2001), Nosal and Wallace (2007) and Choi (2013). Models with divisible assets include Berentsen and Rocheteau (2004), Shao (2009), Sanches and Williamson (2011) and Lester et al. (2012). Related papers with private information about traders' attributes (e.g. preferences) include Faig and Jerez (2006), Ennis (2008) and Dong and Jiang (2010,2013).

produce but give less utility. Sometimes an agent recognizes quality before accepting a good in trade, and sometimes not (it is match specific). Depending on parameters, it is often the case that in equilibrium agents accept goods they do not recognize: Suppose otherwise; then since agents who do recognize low-quality goods never accept them, agents with "lemons" cannot trade; therefore no one produces "lemons" and hence you can accept goods with impunity even if you cannot evaluate their quality before trading. In this situation, equilibrium entails mixed strategies, where sellers produce low quality, and buyers accept unrecognized goods, with positive probabilities. Fiat currency can improve welfare, because in some monetary equilibria the incentive to produce high quality goes up.

Moving to private information about asset quality, consider our benchmark economy with one-period-lived assets in fixed supply $A$, with payoff $\rho \in \{\rho_L, \rho_H\}$, where $\Pr(\rho = \rho_H) = \zeta$ and $\Pr(\rho = \rho_L) = 1 - \zeta$. Assume $\rho$ is common to all units of the asset held by an agent, so it cannot be diversified, but it is independent across agents – e.g., the payoff depends on local conditions specific to the holder. The asset holder has private information about $\rho$ (as in Plantin 2009). Suppose the holder in a meeting makes a take-it-or-leave-it offer. Using Cho and Kreps' (1987) refinement, Rocheteau (2011) shows the equilibrium is separating: holders of $L$-type assets make the full-information offer, $c(q_L) = \min\{\rho_L a, c(q^*)\}$ and $d_L = c(q_L)/\rho_L$, while holders of $H$-type assets make an offer satisfying

$$\rho_H d_H \geq c(q_H) \tag{45}$$

$$u(q_H) - \rho_L d_H \leq u(q_L) - c(q_L). \tag{46}$$

In particular, (46) says $L$-type buyers have no incentive to offer $(q_H, d_H)$.

The least-cost separating offer satisfies (45)-(46) at equality, so $d_H = c(q_H)/\rho_H \in (0, d_L)$, while $q_H \in [0, q_L)$ solves $u(q_H) - c(q_H)\rho_L/\rho_H = u(q_L) - c(q_L)$.Thus, $H$-type buyers retain a fraction of their holdings as a way to signal quality, and hence $q_H < q_L$ (interpretable as over-collateralization, as in DeMarzo and Duffie 1999). When the $L$-type asset is a pure "lemon," $\rho_L = 0$, both $q_L$ and $q_H$ tend to 0.[57] In contrast

[57]This is the case in Nosal and Wallace (2007), Shao (2009) and Hu (2013). The equilibrium, however, would be defeated in the sense of Mailath et al. (1993). A perhaps more reasonable outcome is the best pooling equilibrium from the viewpoint of a buyer with the $H$-type asset, where $q$ solves $\max\{u(q) - \rho_H d\}$ st $\zeta \rho_H d = c(q)$; this still implies $q$ is inefficiently low.

to models following Kiyotaki and Moore (1997) by assuming exogenous constraints, here agents turn over all their assets in trade with probability $1-\zeta$, and a fraction $d_H/a$ with probability $\zeta$, where $d_H/a$ depends on $\rho_L$ and $\rho_H$. With a one-period-lived asset, while $\phi^* = \beta\left[\zeta\rho_H + (1-\zeta)\rho_L\right]$ is the "fundamental" value, here we get

$$\phi = \phi^* + \beta\alpha\sigma\theta\left\{\frac{\zeta\rho_H\lambda(q_H)\rho_L\lambda(q_L)}{\rho_H\lambda(q_H)+\rho_H-\rho_L} + (1-\zeta)\rho_L\lambda(q_L)\right\}.$$

The liquidity premium $\phi - \phi^*$ depends on $\rho_H - \rho_L$. As $\rho_L \to \rho_H$, the premium goes to $\beta\alpha\sigma\theta\lambda(q)$. As $\rho_L \to 0$, the asset becomes illiquid and $\phi \to \phi^*$. If the asset is abundant, $A\rho_L \geq c(q^*)$, the liquidity premium is 0 even though $q_H < q_L = q^*$. Thus, $q_H$ can be inefficiently low even with abundant assets when they are imperfectly recognizable, meaning other assets may also play an essential role. In Rocheteau (2008), the second asset is fiat money. If it is perfectly recognizable then the same logic applies: agents with $L$-type assets make the complete-information offer while those with $H$-type assets make an offer that others do not want to imitate. When $\iota \to 0$, agents hold enough currency to buy $q^*$, and no one uses the asset in DM trade. For $\iota > 0$ they do not hold enough currency to buy $q^*$, and spend it all plus a fraction of their risky assets. Asset liquidity as measured by this fraction clearly depends on monetary policy.[58]

Asset quality can also depend on hidden actions. One rendition allows agents to produce assets of low quality, or that are outright worthless, as when through history coins were clipped or notes counterfeited (Sargent and Velde 2003; Mihm 2007; Fung and Shao 2011). Li et al. (2012) have a fixed supply of one-period-lived assets yielding $\rho$. At some fixed cost $\kappa > 0$, agents can produce counterfeits that yield 0. Assume counterfeits are confiscated by the government after each round of DM trade, so they do not circulate across periods. Then with $\theta = 1$, as above, the offer satisfies $c(q) \leq \rho d$

[58] We emphasize the distinction between riskiness and asymmetric information here. Suppose the asset is risky, but buyers and sellers have the same information when they meet (i.e., uncertainty is resolved after trade). The asset then functions perfectly well as a medium of exchange, with actual returns replaced by expected returns. However, suppose they are symmetrically informed but now see the realization of $\rho$ before DM trade. Then risk is reflected in the previous CM price, with a lower liquidity premium. Based on this reasoning, Andolfatto and Martin (2013) and Andolfatto et al. (2014) consider an environment where the asset is risky and information about $\rho$ is available in the DM, prior to trading, at zero cost. They show nondisclosure is generally desirable, because trade based on expected dividends better smooths consumption. This result, however, can be overturned if individuals are able to access hidden information.

and the IC is

$$-(\phi - \phi^*)a + \beta\alpha\sigma\theta\left[u(q) - d\rho\right] \geq -\kappa + \beta\alpha\sigma\theta u(q), \tag{47}$$

where $\phi^* = \beta\rho$. The LHS of (47) is the payoff to accumulating genuine assets, the holding cost plus the DM surplus, while the RHS is the payoff to accumulating counterfeits, cost $\kappa$ plus the DM surplus.

Given $a = d$, in equilibrium, (47) can be rewritten as an upper bound on the amount of asset that can be transferred,

$$d \leq \frac{\kappa}{\phi - \phi^* + \beta\alpha\sigma\theta\rho}. \tag{48}$$

This endogenous upper bound increases with the cost of counterfeiting $\kappa$, while it decreases with the cost of holding assets $\phi - \phi^*$ and the frequency of DM trading opportunities $\alpha\sigma$. If $\alpha\sigma = 1$, so there are no search or matching frictions in the DM, (48) requires simply that the value of the asset $\phi d$ is less than the cost of fraud $\kappa$. As $\kappa \to 0$ the asset stops circulating. Notice also that an increase in the (endogenous) price $\phi$ tightens the resalability constraint, which leads to pecuniary externalities with non-trivial policy implications, as described in Li et al. (2012).

The CM asset price depending on parameters. If (48) binds and $d < A$ the asset is illiquid at the margin and $\phi = \phi^*$. If (48) does not bind the asset is perfectly liquid and $\phi = \phi^* + \beta\alpha\sigma\theta\rho\lambda(q)$ with $c(q) = \min\{A\rho, c(q^*)\}$. There is an intermediate case where (48) binds, $d = A$, and $\phi = \phi^* - \beta\alpha\sigma\theta\rho + \kappa/A$. In this case the liquidity premium, $\kappa/A - \beta\alpha\sigma\theta\rho$, increases with $\kappa$ and decreases with $\beta\alpha\sigma\theta$. Notice the threat of fraud can affect asset prices even if fraud does not occur in equilibrium. The model also explains why assets with identical yields can be priced differently. If the asset subject to fraud is fiat money, as in Li and Rocheteau (2009), then (48) becomes $c(q) \leq \kappa/(\beta\iota + \beta\alpha\sigma\theta)$, another case where acceptability is not exogenous. Other extensions are Li and Rocheteau (2009) who have long-lived assets, Gomis-Porqueras et al. (2014) who have two currencies and study exchange rates. See also Li et al. (2012) and Williamson (2014), Li and Rocheteau (2011) and Shao (2013).[59]

[59] If we have multiple assets, each with cost of fraud $\kappa_j$ and supply $A_j$, then normalizing $\rho_j = 1\ \forall j$, $c(q) = \min\left\{\sum_j \Omega_j A_j, c(q^*)\right\}$ where $\Omega_j = \kappa_j/\eta_1 A_j$ if $\kappa_j < \eta_1 A_j$ and $\Omega_j = 1$ otherwise. Aggregate liquidity is a weighted average of asset stocks, with weights depending asset characteristics, consistent with a definition for the quantity of money suggested by Friedman and Schwartz (1970).

To this point we have assumed that sellers may be uninformed about the quality of buyers' assets. Lester et al. (2012) let sellers pay a cost to better recognize the quality of an asset and hence endogenize the fraction of matches where it is accepted (Kim 1996 and Berentsen and Rocheteau 2004 similarly let agents pay a cost to better recognize the quality of goods). To simplify the analysis, Lester et al. (2012) assume fraudulent assets are worthless and can be produced at no cost, $\rho_L = 0$ and $\kappa = 0$. This implies sellers only accept assets if they recognize them, and so we can use standard bargaining theory – when sellers recognize assets there is no private information; when they don't the assets are not even on the bargaining table.

We now interpret $\sigma$ as the probability of a single coincidence times the probability that the seller can discern, and therefore accept, a buyer's assets. The information technology allows agents to choose $\sigma$ at a cost $C(\sigma)$ at the beginning of the DM, where $C'(\sigma) > 0$ and $C''(\sigma) < 0$ for $\sigma > 0$, with $C'(0) = 0$ and $C'(\bar{\sigma}) = \infty$ for some $\bar{\sigma}$. The decision of a seller to invest resources to become informed, so that he can accept assets in the DM, is similar to a decision to enter market in the first place or the search intensity decision discussed above. In order to give them some incentive to invest, sellers must have some bargaining power, so we use Kalai's solution with $\theta \in (0, 1)$. The FOC is $C'(\sigma) = \alpha\theta_0 [u(q) - c(q)]$, where as always $v(q) = \min\{A\rho, v(q^*)\}$, with $\rho$ the dividend on genuine assets. This equates the marginal cost of becoming informed to the expected benefit from being able to accept assets. As the marginal cost of information decreases, $\sigma$ and $\phi$ increase. If the asset is long lived, there are strategic complementarities between agents' holdings of liquid assets and their information choice, creating the possibility of multiple steady-state equilibria.

Suppose there are two short-lived assets, both yielding $\rho$. Asset 1 is perfectly recognizable at no cost in a fraction $\bar{\sigma}$ of all meetings while asset 2 requires an ex ante investment $C(\sigma_2)$ to be acceptable in a fraction $\sigma_2$ of meetings. As a result, in an endogenous fraction $\sigma_2$ of matches agents can pay with both assets, while in a fraction $\sigma_1 = \bar{\sigma} - \sigma_2$ they can pay only with asset 1. The investment decision satisfies

$$C'(\sigma_2) = \alpha\theta_0 \{[u(q_2) - c(q_2)] - [u(q_1) - c(q_1)]\}. \tag{49}$$

The RHS of (49) is the benefit to the seller from being informed, or the extra surplus generated by having a payment capacity of $(A_1 + A_2)\rho$ instead of $A_1\rho$. Asset prices are $\phi_1 = \phi^* + \beta\alpha\sigma_1\theta\rho\lambda(q_1) + \beta\alpha\sigma_2\theta\rho\lambda(q_2)$ and $\phi_2 = \phi^* + \beta\alpha\sigma_2\theta\rho\lambda(q_2)$. If there is an increase in the supply of recognizable assets $A_1$, agents invest less in information, and asset 2 becomes less liquid. As a result, $\phi_2$ decreases. If the recognizable asset is fiat money, then the acceptability of the other asset is affected by monetary policy. At $\iota = 0$, $q_1 = q_2 = q^*$ and agents stop investing in information, and asset 2 becomes illiquid, reflecting the old idea that the use of money saves information costs.[60]

# 12 Generalized OTC Markets

Papers spurred by Duffie et al. (2005) maintain tractability by restricting $a \in \{0, 1\}$; Lagos and Rocheteau (2009) relax this.[61] For simplicity, suppose all trade goes through dealers, as in Section 6 with $\alpha_D > 0 = \alpha_I$. Let $a \in \mathbb{R}_+$, and assume $u_j(a)$ is the flow utility of an agent with preference type $j \in \{0, ..., I\}$. Each $I$ gets a shock $j \in \{0, ..., I\}$ at Poisson rate $\omega_j$, with $\sum_i \omega_i = \omega$. When $I$ with preference $j$ contacts $D$, they bargain over the $a_j$ that $I$ takes out of the meeting, and a payment that includes $D$'s cost of the transaction $q(a_j - a)$ plus a fee $\varphi_j(a)$. The choice of $a_j$ reduces to $\max_{a'} \{\bar{u}_j(a') - rqa'\}$, where

$$\bar{u}_j(a) \equiv \frac{(r + \eta) u_j(a) + \sum_k \omega_k u_k(a)}{r + \eta + \omega}$$

$\eta$ is the relevant arrival rate. The intermediation fee is

$$\varphi_j(a) = \frac{\theta_D [\bar{u}_j(a_j) - \bar{u}_j(a) - rq(a_j - a)]}{r + \eta}. \tag{50}$$

Equilibrium is given by the desired asset positions $(a_i, ...a_I)$, the fee $\varphi_j(a)$, the $q$ that clears the interdealer market, and a distribution $(n_{ij})$ satisfying the usual conditions. Here we focus on implications for asset prices and measures of liquidity. Let $u_i(a) = \psi_i \log a$, where $\psi_1 < \psi_2 < ... < \psi_I$, and let $\bar{\psi} = \sum_j \omega_j \psi_j / \omega$. Then individual asset demand is

$$a_j = \frac{(r + \eta) a_j^{\infty} + \omega \bar{a}}{r + \eta + \omega}, \tag{51}$$

[60] Lester et al. (2012) and Zhang (2013) use versions of this model to study international issues. Hu and Rocheteau (2014) use mechanism design, while Dong (2009), Nosal and Rocheteau (2011) and Lotz and Zhang (2013) study money and credit. While this work focuses on information, other aproaches include Zhu and Wallace (2007), Nosal and Rocheteau (2013) and Hu and Rocheteau (2013). See also Kocherlakota (2003), Andolfatto (2011) and Jacquet and Tan (2012).

[61] In spirit if not detail, the idea is to do what Section 7 does for monetary theory: extend second-generation theory to a more general yet still tractable framework. The new model captures aspects of illiquid markets like participants adjusting positions to reduce trading needs.

where $a_j^\infty = \psi_j / rq$ would be demand in a frictionless market, and $\bar{a} = \bar{\psi}/rq$ is demand from $I$ with average valuation $\bar{\psi}$. In a frictional market, $I$ chooses holdings between $a_j^\infty$ and $\bar{a}$, with the weight assigned to $a_j^\infty$ increasing in $\eta$. If $I$ can readjust $a$ more frequently, they choose a position closer to the frictionless case.

Hence, frictions concentrate the asset distribution. As frictions decrease, $a_j \to a_j^\infty$ and the distribution becomes more disperse, but aggregate demand is unchanged. A message is that one should not expect to identify frictions by looking at prices alone. Trade volume is $\alpha_D \sum_j n_{ij} |a_j - a_i| / 2$, lower than in a frictionless economy. It increases with $\eta$, capturing the idea that large volume characterizes liquid markets, where $I$ can switch in and out of positions easily. Frictions have a direct plus indirect effects on volume. If $I$ finds trading opportunities more frequently, the measure of $I$ that can trade rises, but the measure that are mismatched falls. An increase in $\alpha_D$ shifts the distribution so that volume tends to increase. In contrast to Duffie et al. (2005), where $a \in \{0, 1\}$, now volume decreases with $\theta_D$.[62]

Another measure of liquidity is the bid-ask spread or intermediation fee. In Duffie et al. (2005), an increase in $\alpha_D$ raises $I$'s value of search for alternative traders, so spreads narrow. Now spreads still depend on $\alpha_D$, but also on the extent of mismatch between asset positions and valuations. Hence, the relationship between the spread and $\eta$ can be nonmonotone: under reasonable conditions, one can show the spread vanishes as $\eta \to 0$ or as $\eta \to \infty$. In liquid markets, $I$ has good outside options, and hence $\varphi$ is small; in illiquid markets, $I$ trades very little, so $a_j^\infty$ is close to $A$, and $\varphi$ is small. The model also predicts a distribution of transactions, with spreads increasing in the size of a trade, as well as varying with $I$'s valuation.[63] The model can also be extended to allow heterogeneity across $I$ in terms of arrival rates or bargaining power, and those with higher $\eta$ trade larger quantities at a lower cost per unit.

Trading delays are an integral feature of the microstructure in OTC markets. The time it takes to execute a trade not only influences volume and spreads, but is often used directly as a measure of liquidity. Lagos and Rocheteau (2007,2008*b*) endogenize $\alpha_D$ with entry by $D$, and derive some new predictions, including a change

[62] Branzoli (2013) estimates a variant of this model using data from the municipal bond market. He finds $\theta_D$ is sufficiently high to reduce trade volume by 65% to 70%.

[63] The relationship between spread and trade size generally depends on details. Lester et al. (2013) find with competitive search and a Leontief matching technology, costs decrease with the size of the trade, in accordance with evidence from corporate bond markets.

in the equilibrium set due to the nonmonotonicity mentioned above. The model can generate multiple equilibria: it may be illiquid because participation by $D$ is low given a belief that $I$ will only trade small quantities; and it is rational for $I$ take conservative positions given long trading delays. Tight spreads are correlated with large volume and short delays across equilibria, and scarce liquidity can arise as a self-fulfilling prophecy. Subsidizing entry can eliminate this multiplicity. Even when equilibrium is unique, the model has novel predictions, like lower market power for dealers promoting entry and reducing delays by increases trade size. Similarly, a regulatory reform or a technological innovation that gives $I$ more direct access to the market (e.g., Electronic Communication Networks) implies an increase in market liquidity and intermediated trade.

The model also provides insights on welfare in illiquid markets. At least when contact rates are exogenous, in Duffie et al. (2005) welfare is unaffected by $\theta_D$, which only affects transfers from $I$ to $D$. When $a$ is not restricted to $\{0, 1\}$, equilibrium is inefficient unless $\theta_D = 0$. Indeed, if $D$ captures any of the gains that $I$ gets from adjusting his portfolio, $I$ economizes on intermediation fees by choosing $a_j$ closer to $\bar{a}$, thus increasing mismatch. When $\alpha_D$ is endogenous, the equilibrium is generically inefficient, again related to Hosios (1990). Efficient entry requires $\alpha_D$ equal the contribution of dealers to the matching process, but efficiency along the intensive margin requires $\alpha_D = 0$. As in monetary models, those two requirements are incompatible, and the market is inefficient, although as in many other models, a competitive search version can deliver efficiency (Lester et al. 2013).[64]

When $\alpha_I = 0 < \alpha_D$, investors trade only with dealers who continuously manage positions in a frictionless market. As mentioned earlier, some markets are well approximated by this, while others are better represented by $\alpha_I > 0$, such as the Federal Funds market, where commercial banks trade overnight loans typically without intermediation. Afonso and Lagos (2014$a$) model this explicitly. Banks have $a \in \mathbb{R}_+$ and trade bilaterally, and so there is a distribution $F(a)$. Afonso and Lagos (2014$b$) consider a special case with $a \in \{0, 1, 2\}$, where equilibrium is similar to some of the monetary models above. More generally, Afonso and Lagos (2014$a$) show existence and uniqueness, characterize the terms of trade, and address various positive and

[64] Branzoli (2013) finds that the most effective way to promote trading activity is the introduction of weekly auctions where investors can trade bilaterally.

normative questions, including quantitative questions facing central banks.

In terms of other applications and extensions, Biais et al. (2014) give a reinterpretation where agents have continuous access to the market, but learn their valuations infrequently. They also provide a class of utility functions nesting Duffie et al. (2005) and Lagos and Rocheteau (2009). Lagos and Rocheteau (2008) allow investors to have both infrequent access to the market, where they are price-takers, and to dealers, where they bargain. Pagnotta and Philippon (2012) use a similar model to study marketplaces competing on speed. Melin (2012) has two types of assets traded in different markets, one with search and one frictionless. Mattesini and Nosal (2013), Geromichalos and Herrenbreuck (2013) and Lagos and Zhang (2013) integrate generalized models of OTC markets with the monetary models in Section 7.

In particular, in Lagos and Zhang (2013) an asset $a$ called equity with dividend $\rho$ is held by $I$ with time-varying idiosyncratic valuations. There are gains from trade in $a$ from heterogeneous valuations, and $I$ participate in an OTC market like the DM in our benchmark monetary model, but intermediated by $D$, who again has access to a frictionless interdealer market. In this market fiat money is essential as a medium of exchange, and as usual some mechanism like bargaining determines the terms of trade between $I$ and $D$. The DM alternates with a frictionless CM where agents rebalance portfolios. Equilibrium entails a cutoff preference type such that $I$ below this who meets $D$ trades all his equity for cash and $I$ above this trades all his cash for equity.[65]

When $I$ has all the bargaining power, he trades equity for cash at the price in the interdealer market. If $D$ has all the bargaining power, $I$ trades at a price higher (lower) than the interdealer market if $I$ wants to buy (sell) assets. This implies a bid-ask spread determined by monetary policy, and details of the market structure, such as the speed at which $I$ contacts $D$ or bargaining power. A nonmonetary equilibrium always exists, and when $\pi$ is large it is the only equilibrium. In nonmonetary equilibrium only $I$ holds equity, since there is no trade in the OTC market, and the equity price is the expected discounted value of the dividend stream for the average $I$. Monetary equilibrium exists for lower $\pi$. For $\pi$ not much larger than $\beta - 1$, there

[65] It would not be difficult to incorporate other assets or consider situations where agents obtain money loans collateralized with assets. Basically, any of the extensions in the monetary models discussed above can be incorporated.

is a unique stationary monetary equilibrium where $D$ holds all the equity overnight, while $I$ holds it intraday. The asset price is higher than in nonmonetary equilibrium, because OTC exchange props up the resale value of assets. As usual, the Friedman rule implements the efficient allocation, and real asset prices decrease with $\pi$ because money is complementary with (used to purchase) $a$. With entry by $D$, Lagos and Zhang (2013) generate sunspot equilibria with recurrent episodes that resemble financial crises – when a sunspot shock hits, spreads spike, while volume, trading frequency, market-making activity and asset prices collapse. As in many of the models presented above, this is driven by the self-referential nature of liquidity.

Another area where New Monetarist models can provide useful and practical advice, somewhat related to the above-mentioned work on Fed Funds, concerns payment and settlement systems in general. Central banks around the world oversee such systems, e.g., the Division of Reserve Bank Operations and Payment Systems oversees the Federal Reserve Banks as providers of financial services to depository institutions and fiscal agency services to the government. The cashless New Keynesian framework, standard GE theory, or reduced form (e.g., CIA or MUF) models cannot address the important issues faced by the regulators of these systems, where trade seems better characterized as bilateral. Freeman (1996) is an early effort at formally modeling this activity. Koeppl et al. (2008), Williamson and Wright (2010*b*, Sec. 5) and Nosal and Rocheteau (2011, Ch. 9) present versions based on our benchmark model from Section 7. See McAndrews et al. (2011) for a survey of related work and a general discussion of the issues.

# 13 Conclusion

The literature summarized in this survey helps us understand economic phenomena at many levels and covers much territory, from rudimentary theories of commodity money to fully-articulated quantitative macro systems to microstructure depictions of Fed Funds trading. While they share some features, the models also vary a lot. Some are relatively "clean" and have few hidden assumptions about the environment or presumptions about behavior or institutions. This comes at a cost, and may entail using primitive formulations. Other models are more sophisticated, and less "clean," because they are designed more for applications. *This is not to say policy or empirical* *work should use inferior theory.* But there are tradeoffs, and finding the right balance is part of the art of monetary economics. As with most art, what constitutes good work is partly subjective, but technique matters, and it seems to us that much current practice uses poor technique.[66]

It is poor technique to tell agents in a model to do something when there are profitable deviations, as when they are assumed to make trades that are not in the pairwise core because of an exogenous payment (e.g., CIA) constraint. We also think it makes little sense to tell agents to take prices as given when prices are wrong due to nominal rigidities. An artless claim would be that this is no different than imposing restrictions on what they can do by specifying the environment in a particular way. We disagree. Imposing frictions in the physical environment, like spatial separation, limited commitment or imperfect information, has all kinds of implications for resource and incentive feasibility. The way to proceed is to lay out all of these restrictions and draw out all their implications. This does not mean we necessarily get the first best, since we must respect genuine frictions. But shortcuts like CIA constraints are *not* the same as having genuine frictions in the environment; they are impositions on behavior. We do not think the distinction is subtle.

We do not endorse the use of models with fiat currency in utility or production functions. From a theoretical perspective, it means giving up on interesting and challenging questions. From a policy perspective, it is all too obviously subject to the Lucas critique. Of course, as with any specification, the appropriateness depends on the issues at hand, but it is hard to imagine why anyone would *prefer* reduced-form models. We can imagine hearing "this is the best we can do" – to which we suggest "read the papers summarized above." This does not mean assets in general cannot appear in utility or production functions. Productive capital, housing, wine and art belong there. Theories presented above take as primitives objects that may give direct utility from consumption, like wine, or flow dividends, like art, and then ask if and how they get used in the facilitation of exchange. Fiat money is special because it provides no direct utility. But to study liquidity, when evaluating any

[66] Further discussion of reduced-form monetary theory follows below. As for the "cashless" New Keynesian model, which attempts to avoid the charge of practicing poor monetary theory by ignoring the issues, their downside is that they are unable to address virtually all the topics covered in this essay, and that is perhaps a more serious indictment. Again, this is partly subjective, but it seems incumbent upon us in this forum to profess our convictions.

asset, one should take into account its role in the exchange process. This necessitates modeling the exchange process.

Since assets can show up in indirect utility or value functions, logically, it is difficult to out-perform reduced-form models: whatever we adopt for primitives, the same results (at least for a given set of observables) obtain if one starts with $V(\cdot)$ and calls that the utility of assets. The model in Section 7 is observationally equivalent (for a given set of observables) to a MUF model with utility $U(x) - \ell + \beta V(\hat{a})$, or a CIA model with the constraint plugged in. But how does one know ex ante how to specify $V(\cdot)$? Rather than guess, we *derived*

$$V(a) = W(a) + \alpha\sigma[u(q) - v(q)], \tag{52}$$

where key ingredients are $\alpha$ and $\sigma$, capturing search and matching, and $v(q)$, nesting a general class of mechanisms. With heterogeneity due to either primitives, or to histories, as in Molico (2006), the distribution $F(a)$ is another important ingredient, because in models where agents trade with each other it determines trading partners, and not just the slope of the budget line. With private information, $\chi$ also shows up, looking like a pledgeability parameter in Kiyotaki-Moore (1997), except it is endogenous, depending on policy, and can impact trade on intensive and extensive margins (Lester et al. 2012; Li et al. 2012).

These key ingredients – $\alpha$, $\sigma$, $v$, $F$ and $\chi$ – open up new avenues of theoretical exploration, provide new insights into quantitative issues, and change our priors on important policy questions. On inflation, in particular, the models presented above incorporate a variety of effects: (i) inflation is a tax on real balances; (ii) some pricing mechanisms, like bargaining, can compound this wedge; (iii) there are distortions on the intensive and extensive margins, the latter revolving around participation decisions, search intensity, congestion externalities and reservation trade decisions; (iv) nominal rigidities, signal extraction problems, private information and related elements can be incorporated; and (v) there can be significant effects of the distribution of liquidity and the distribution of prices. As compared to popular reduced-form models, we learn a lot from considering ingredients they do not have. Other policy issues that are affected by similar considerations include the desirability of low nominal interest rates (they are typically good, but there are exceptions), the wisdom of trying to reduce unemployment using expansionary policy (this can be counterproductive, and even if feasible it can be undesirable), and the interpretation of sticky prices in the data (they do not logically rationalize Keynesian theory or policy).

While the constraints in some of our models may "look like" CIA constraints, as we said, they are really feasibility constraints. In some versions, like Kiyotaki-Wright (1989), it is obvious that resource-feasibility restrictions imply one cannot turn over something one does not have, and this is *not* a CIA constraint – indeed, the baseline model does not even have cash. There are also incentive-feasibility restrictions that need to be considered. In some models, you cannot issue an IOU if the other party thinks you would renege because of commitment/enforcement limitations, and you cannot easily use an asset for payment (or collateral) if they think it might be a "lemon." This does not come up in CIA models, where IOU's and the use of other assets are ruled out by fiat. The models here try to be explicit about the frictions that make credit difficult, with Kocherlakota (1998) being a leading example, and Kehoe-Livine (1993) or Kiyotaki-Moore (1997) also providing insight. Extensions of our baseline model with multiple assets make it transparent that one does not need to impose ad hoc restrictions on their use in transactions because this can be derived endogenously. Which assets get used and how? A challenge is to look this (what Helwig 1993 calls the modified Hahn problem) in the face, rather than evade it.

Based on this, we dispute the insinuation that search-and-bargaining models are "the same as" CIA or MUF models (Camera and Chien 2013). Compared to (52), they miss $\alpha\sigma$, which is especially important when endogenized via entry, intensity or reservation decisions that are not invariant to interventions. The reduced-form models we know also miss $v(q)$ by sticking to price taking behavior. Could one add elements like $\alpha\sigma$, $\chi$, $v(\cdot)$ or $F(\cdot)$ to those models? Sure. Then they would be search-and-bargaining models. However, reduced-form theory still has fundamental problems – e.g., it is not amenable to mechanism design, which is an important tool for assessing the role of money and related institutions. But labels aside, an insight that is obvious with the benefit of hindsight is that once one has such a specification one does not need as hoc devices like CIA or MUF. A concrete example is Diamond (1984), a version of Diamond (1982) with a CIA constraint. As one can learn from Kiyotaki and Wright (1991,1993), in a very similar environment, once one thinks a little more about specialization, which makes barter clumsy, as well as commitment and information, which make credit untenable, one discovers there is a role for money in this environment *without* a CIA constraint.

Although search per se is not critical, as we said several times, it can be convenient for capturing the difficulty of barter and credit. Bargaining is also not crucial, although it may be appropriate in some applications. Berentsen et al. (2011), e.g., in accounting for the behavior of unemployment and inflation, argue that both search and bargaining are important for qualitative and quantitative results. Bargaining considerations can be also quantitatively relevant for understanding the effects of monetary policy on welfare, employment and investment. But for other applications, other mechanisms are preferred, e.g., implementation of sticky-price theory microfounded on Burdett-Judd (1982) pricing. Relatedly, Lagos (2010*b*) shows that models with trading frictions help us understand issues in financial economics, like the equity-premium or risk-free-rate puzzle, quantitatively. More generally, with reasonable parameter values the models can generate equilibria that appear anomalous from the vantage of standard asset-pricing theory, which we take to mean that finance has something to learn from monetary economics (Rocheteau and Wright 2013). The models also shed new light on optimal policy (Aruoba and Chugh 2008), banking (Berentsen et al. 2011; Gu et al. 2013*a*), OTC markets (Duffie et al. 2011; Lagos and Rocheteau 2009), and both conventional and unconventional monetary policy (Williamson 2012,2014).

There is more research on related issues and models, but we have to stop somewhere. So here it is. Our hope is that readers will continue to search for developments in the area, and that they have enjoyed – or at least survived – this guided tour.

# Appendix on Notation (not necessarily for publication)

$\alpha$, $r$, $\beta$ = arrival rate, discount rate, discount factor
$\rho$ = (utility of) dividend if $\rho > 0$ or storage cost if $\rho < 0$
$a, A$ = individual, aggregate asset holdings
$\sigma$, $\delta$ = single- and double-coincidence prob
$\kappa$, $\varepsilon$ = cost and probability of entry a la Pissarides
$n_j$ = measure of type $j$, $m_i$ = measure inventory $i$
$\mu$ = monitoring probability
$n$ = fraction of monitored agents in CW
$\tau$ = trading strategy
$\tau = \Upsilon(\tilde{\tau})$ = best response correspondence
$u$, $c$ = utility, cost of DM good
$q$, $Q$ = quantity in monetary, barter trades
$V^A$, $V^B$, $V^C$, $V^D$ value fn for autarky, barter, credit and deviation
$V_a$ = value fn for $a \in \{0, 1\}$
$\theta_a$ = bargaining power of agent with $a \in \{0, 1\}$
$v(q)$ = cost of $q$ – i.e., a general mechanism
$\eta = \alpha\theta$ = arrive rate times bargaining power
$P$, $M$, $C$ = producer, middleman and consumer in RW
$I$, $D$ = investor and dealer in DGP
$S_I$ = surplus in DGP
$\omega_j$ = probability of preference shock $j$ in DGP
$F(a)$ = asset dist'n
$V(a)$, $W(a)$ = value fn for $a \in \mathbb{R}_+$ in DM, CM
$\phi$, $z = (\phi + \rho) a$ = price and value of $a$
$U(x) - \ell$ = CM utility
$S(q) = u(q) - c(q)$ = DM surplus
$q$, $q_b$ = DM quantity in money and in barter trades
$d$, $p = d/q$ = DM dollars and price
$\pi$, $\iota$ = money growth (or inflation) and nominal interest rate
$G$, $T$, $O$ = gov't consumption, transfers and other income in CM
$\lambda$ = liquidity premium or Lagrange multiplier
$w_\ell$, $w_k$ = factor prices for labor and capital
$t_\ell$, $t_k$ = tax rates on labor and capital income
$\Delta$ = depreciation rate on $k$
$\gamma$, $\Gamma$ = DM utility fn, $\Gamma q^{1-\gamma}/(1-\gamma)$
$D$, $\chi$ = KM debt and haircut parameter
$\xi_b$, $\xi_b$ = wedges on shares and bonds in Lagos JME
$R = 1 + \iota$ = gross returns
$\psi$ = DM preference shock, $\psi u(q)$
$\zeta$, $1 - \zeta$ = probability that $\rho = \rho_H$, $\rho = \rho_L$

# Appendix on Commodity Money

To explain the SS condition in Section 2, consider type 1 and pure strategies. If 1 has good 2, he can switch to good 3 only by trading with a 2 that has good 3 (since 3 never has good 3). For this, 1 has to meet 2 with good 3, which occurs with probability $n_2 m_2$, then trade, which occurs with probability $\tau_1$ (since 2 always wants good 2). And if 1 has good 3, he switches to good 2 only by acquiring good 1, consuming and producing a new good 2 (he never switches from good 3 to 2 directly, since if he preferred good 2 he would not trade it for good 3 in the first place). He trades good 3 for good 1 either by trading with 3 that has good 1, which occurs with probability $n_3 m_3$, or with 2 that has good 1 but prefers good 3, which occurs with probability $n_2(1-m_2)(1-\tau_2)$, but we can ignore that since either $\tau_2 = 1$ or $m_2 = 1$. Equating the measure of type 1 that switch from good 2 to good 3 and the measure that switch back, we get (1).

| case | $\tau$ | $m$ | existence? |
|---|---|---|---|
| 1 | $(1,1,1)$ | $(1/2,1/2,1/2)$ | never |
| 2 | $(1,1,0)$ | $\left(\sqrt{2}/2,\sqrt{2}-1,1\right)$ | maybe |
| 3 | $(1,0,1)$ | $\left(\sqrt{2}-1,1,\sqrt{2}/2\right)$ | never |
| 4 | $(1,0,0)$ | $(1/2,1,1)$ | never |
| 5 | $(0,1,1)$ | $\left(1,\sqrt{2}/2,\sqrt{2}-1\right)$ | maybe |
| 6 | $(0,1,0)$ | $(1,1/2,1)$ | maybe |
| 7 | $(0,0,1)$ | $(1,1,1/2)$ | never |
| 8 | $(0,0,0)$ | $(1,1,1)$ | never |

Table 1: Candidate Equilibria in the Commodity Money Model

Table 1 lists candidate equilibria, with existence results for the case $n_j = 1/3$. Consider case 1. After inserting $m$, the BR conditions reduce to

$$\begin{aligned}
\Delta_1 &\geq 0, \text{ or } \rho_3 - \rho_2 \geq u/6 \\
\Delta_2 &\geq 0, \text{ or } \rho_1 - \rho_3 \geq u/6 \\
\Delta_3 &\geq 0, \text{ or } \rho_2 - \rho_1 \geq u/6.
\end{aligned}$$

Since these cannot all hold, this is never an equilibrium. In contrast, for case 2, the BR conditions reduce to

$$\begin{aligned}
\Delta_1 &\geq 0, \text{ or } \rho_3 - \rho_2 \geq (1-\sqrt{2})u/3 \\
\Delta_2 &\geq 0, \text{ or } \rho_1 - \rho_3 \geq 0 \\
\Delta_3 &\leq 0, \text{ or } \rho_2 - \rho_1 \leq (1-\sqrt{2}/2)u/3.
\end{aligned}$$

For some parameters, these all hold and this is an equilibrium. The rest are similar.

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